GRAPHICAL AND MECHANICAL COMPUTATION

PART I. ALIGNMENT CHARTS

BY

JOSEPH LIPKA, PH.D.

ASSISTANT PROFESSOR OF MATHEMATICS IN THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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PREFACE

This book embodies a course given by the writer for a number of years in the Mathematical Laboratory of the Massachusetts Institute of Technology. It is designed as an aid in the solution of a large number of problems which the engineer, as well as the student of engineering, meets in his work.

In the opening chapter, the construction of scales naturally leads to a discussion of the principles upon which the construction of various slide rules is based. The second chapter develops the principles of a network of scales, showing their application to the use of various kinds of coordinate paper and to the charting of equations in three variables.

Engineers have recognized for a long time the value of graphical charts in lessening the labor of computation. Among the charts devised none are so rapidly constructed nor so easily read as the charts of the alignment or nomographic type — a type which has been most fully developed by Professor M. d'Ocagne of Paris. Chapters III, IV, and V aim to give a systematic development of the construction of alignment charts; the methods are fully illustrated by charts for a large number of well-known engineering formulas. It is the writer's hope that the simple mathematical treatment employed in these chapters will serve to make the engineering profession more widely acquainted with this time and labor saving device.

Many formulas in the engineering sciences are empirical, and the value of many scientific and technical investigations is enhanced by the discovery of the laws connecting the results. Chapter VI is concerned with the fitting of equations to empirical data. Chapter VII considers the case where the data are periodic, as in alternating currents and voltages, sound waves, etc., and gives numerical, graphical, and mechanical methods for determining the constants in the equation.

When empirical formulas cannot be fitted to the experimental data, these data may still be efficiently handled for purposes of further computation, — interpolation, differentiation, and integration, — by the numerical, graphical, and mechanical methods developed in the last two chapters.

Numerous illustrative examples are worked throughout the text, and a large number of exercises for the student is given at the end of each chapter. The additional charts at the back of the book will serve

as an aid in the construction of alignment charts. Bibliographical references will be found in the footnotes.

The writer wishes to express his indebtedness for valuable data to the members of the engineering departments of the Massachusetts Institute of Technology, and to various mathematical and engineering publications. He owes the idea of a Mathematical Laboratory to Professor E. T. Whittaker of the University of Edinburgh. He is especially indebted to Capt. H. M. Brayton, U. S. A., a former student, for his valuable suggestions and for his untiring efforts in designing a large number of the alignment charts. Above all he is most grateful to his wife for her assistance in the revision of the manuscript and the reading of the proof, and for her constant encouragement which has greatly lightened the labor of writing the book.

JOSEPH LIPKA.

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CHAPTER I.

SCALES AND THE SLIDE RULE.

1. Definition of a scale. — A graphical scale is a curve or axis on which are marked a series of points or strokes corresponding in order to a set of numbers arranged in order of magnitude.

If the distances between successive strokes are equal, the scale is said to be *uniform* (Figs. 1*a*, 1*b*). If the distances between successive strokes are unequal, the scale is said to be *non-uniform* (Fig. 1*c*). The strokes are drawn as fine as possible, perpendicular to the axis which carries the scale.

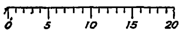


FIG. 1*a*.

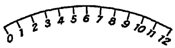


FIG. 1*b*.

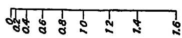
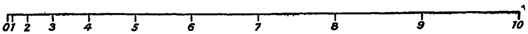


FIG. 1*c*.

2. Representation of a function by a scale. — Consider the function u^2 of a variable u. Form the table

$u =$	0	1	2	3	4	5	6	7	8	9	10
$u^2 =$	0	1	4	9	16	25	36	49	64	81	100

and on an axis OX lay off from the origin O, lengths equal to $x = 0.04\,u^2$ inches (Fig. 2*a*); mark at the strokes indicating the end of each segment the corresponding value of u. Thus, a stroke marked u is at a distance of $0.04\,u^2$ inches from the origin. Fig. 2*a* is said to represent the function u^2 by a scale. The length 0.04 inches is chosen arbitrarily in this case to represent the unit segment used in laying off the values of u^2 on the axis. This unit segment is called the *scale modulus*.



(*u*)

FIG. 2*a*.

In general, any function $f(u)$ of a variable u such that each value of the variable determines a single value of the function, may be represented by a scale. Form the table

$u =$	u_1	u_2	$u_3 \ldots$
$f(u) =$	$f(u_1)$	$f(u_2)$	$f(u_3) \ldots$

and on an axis OX lay off from the origin lengths equal to $x = mf(u)$ inches, and mark with the corresponding values of u the strokes indicating the end of each segment. Fig. 2*b* is said to represent the function $f(u)$ by a scale. The length m inches is chosen arbitrarily to represent the unit segment used in laying off the values of $f(u)$, and it is called the *scale modulus.* The equation $x = mf(u)$ is called the *equation of the scale.*

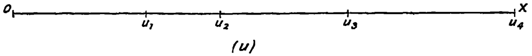


FIG. 2*b*.

The *uniform scale* is a special case of this representation when $f(u) = u$. In Fig. 2*c*, $x = mu$, where $m = 0.5$ inches.*

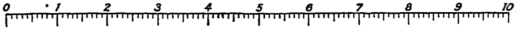


FIG. 2*c*.

The *logarithmic scale* is a special case of this representation when $f(u) = \log u$. In Fig. 2*d*, $x = m \log u$, where $m = 12.5$ cm.*

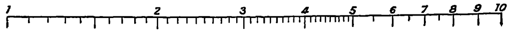


FIG. 2*d*.

The uniform and logarithmic scales are the most important scales for our work.

After we have constructed a scale for $f(u)$ from a table of values of u and $f(u)$, we may wish to estimate the value of u corresponding to a stroke intermediate between two strokes of the scale, or to estimate on the scale the position of a stroke corresponding to a value of u intermediate between two values of u in the table. This process of *interpolating on the scale* is of course very much easier for uniform scales than for non-uniform scales. The accuracy of such interpolation evidently depends upon the interval between two successive strokes. Experience has shown that this interval should not be less than 1 mm. or about $\frac{1}{20}$ in. (very rarely need it be as small as this); this may always be done by the choice of a proper scale modulus.

3. **Variation of the scale modulus.** — By varying the modulus m with which a scale for $f(u)$ is constructed, we get a series of scales $x_1 = m_1 f(u)$, $x_2 = m_2 f(u)$, $x_3 = m_3 f(u)$, . . . , all representing the same

* The values of m given in the text are those which were employed originally in the construction of the scales; these values do not however refer to the cuts which, in most cases, are reductions of the original drawings.

function with moduli m_1, m_2, m_3, . . . , respectively. Only one of these scales need be constructed by means of a table of values of $f(u)$, and the others may be derived graphically from this.

In Fig. 3a, let O_1X_1 carry the scale $x_1 = m_1f(u)$; we wish to construct the scale $x_2 = m_2f(u)$. Let O be any convenient point; join OO_1 and on this line choose O_2 such that $OO_2/OO_1 = m_2/m_1$; through O_2 draw O_2X_2 parallel to O_1X_1. If A is a point on O_1X_1 marked u', then $O_1A = m_1f(u')$, and OA will cut O_2X_2 in a point B such that $O_2B/O_1A = OO_2/OO_1$ or $O_2B = \frac{m_2}{m_1} \cdot m_1f(u') = m_2f(u')$, and thus B will also be marked u'. By joining O with all the points A of the scale O_1X_1 we shall thus get the points B of the scale O_2X_2 so that corresponding points on the same

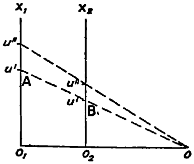


FIG. 3a.

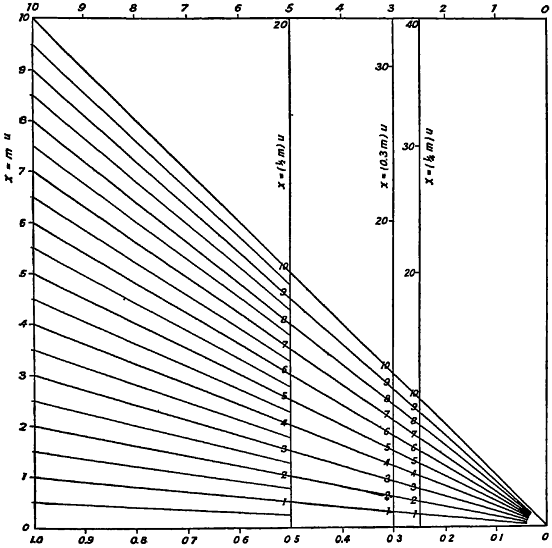


FIG. 3b.

transversal through O will be marked with the same value of u, and the scale on O_2X_2 will have for its equation $x_2 = m_2f(u)$.

The transversals through O need not be drawn, but simply their points of intersection with O_2X_2 indicated. If the transversals through O are drawn, then we may get a scale of any required modulus by merely drawing a parallel to O_1X_1 dividing the segment OO_1 in the required ratio; thus a line midway between O and O_1 will carry a scale with modulus

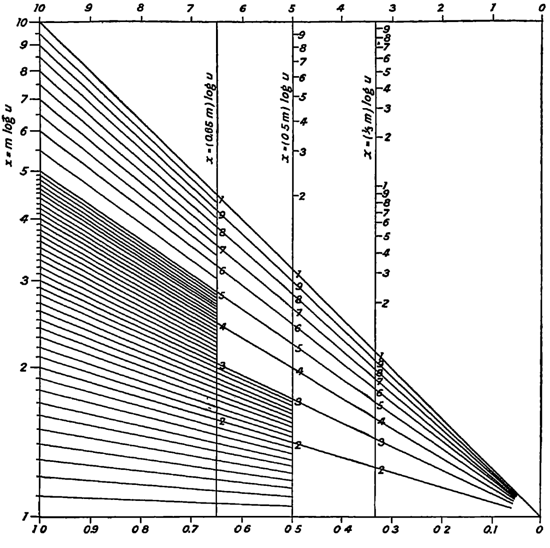


FIG. 3c.[1]

$m_1/2$, a line $\frac{2}{5}$ of the way from O to O_1 will carry a scale with modulus $2\, m_1/5$, etc.

This principle is illustrated in Figs. 3*b* and 3*c* for uniform and logarithmic scales respectively. If we mark a uniform scale .1, .2, .3, ...9, on the base line beginning at O, then the lines through these points parallel to the left-hand scale with modulus m will cut the transversals in scales whose moduli are .1 m, .2 m, . . . , .9 m, respectively. It is best to make the charts in these figures almost square, and to take $m = 10$

in. for the uniform scale and $m = 25$ cm. for the logarithmic scale. The chart of uniform scales will then be an amplification of the engineer's or architect's hexagonal scale, and the chart of logarithmic scales, an amplification of the logarithmic slide rule.

If necessary the scales in either chart may be extended. Note, however, that in the case of the logarithmic scales, the segment representing the interval from $u = 1$ to $u = 10$ is of the same length as the segment representing the interval from $u = 10$ to $u = 100$, or, in general, the segment representing the interval from $u = 10^n$ to $u = 10^{n+1}$.

It is convenient to draw Figs. 3*b* and 3*c* on durable paper. Only the primary scale with modulus m, the base line and the transversals need be drawn. The paper may then be creased along any parallel to the primary scale to give a scale of the required modulus. Charts of this nature have been used to assist in constructing a large number of the scales and charts that follow, and much time and energy have been saved thereby. (Such charts will be found in the back of this book.)

4. **Stationary scales.** — A relation between two variables u and v of the form $v = f(u)$ may be represented graphically by constructing the two scales $x = mv$ and $x = mf(u)$ on opposite sides of the same axis or on adjacent or parallel axes with the same modulus and from the same origin or with origins in a line perpendicular to the axes.

If C represents degrees Centigrade and F represents degrees Fahrenheit, then $F - 32 = 1.8\ C$. We construct the uniform scales $x = m\ (F - 32)$ and $x = m\ (1.8\ C)$ on opposite sides of the same axis, and from the same origin, *i.e*, the points marked $C = 0$ and $F = 32$ coincide. In Fig. 4*a*,

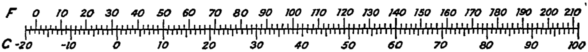


Fig. 4*a*.

$m = 0.02$ in., so that the equations of our scales are $x = 0.02\ (F - 32)$ and $x = 0.036\ C$. By means of such a figure, we may immediately convert degrees Centigrade and Fahrenheit into each other.

If pressure is expressed as pounds per sq. in., P, and feet of water, W, then $P = 0.434\ W$. Draw the uniform scales $x = mP$ and $x = m\ (0.434\ W)$ from the same origin. In Fig. 4*b*, the scale modulus is 1 in., so that we have the scales $x = P$ and $x = 0.434\ W$. We may add another scale for pressure expressed in inches of mercury, M; thus

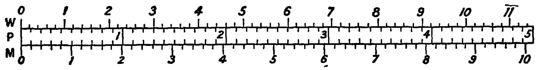


FIG. 4*b*

$P = 0.492\ M$, and the M scale has for equation $x = 0.492\ M$. By means of such a figure (drawn with the aid of chart $3b$ or a pair of dividers) pressure may be read immediately in pounds per sq. in., feet of water, or inches of mercury.

If the relation between the two variables is of the form $v = \log u$, we construct the adjacent scales $x = mv$ and $x = m \log u$. If we take $m = 25$ cm., the logarithmic scale will be the same length as that of the logarithmic slide rule, and if the uniform scale is divided into 500 parts, we can use such a figure to read easily the values of the mantissas of the logarithms of all numbers to three decimals, and conversely to read the numbers corresponding to given mantissas (Fig. $4c$). The slide rule

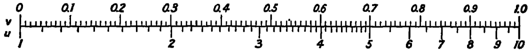


FIG. $4c$.

contains two such adjacent scales. The chart, Fig. $3c$, could be used for the same purpose if we construct a uniform scale adjacent to the primary logarithmic scale.

If the relation between the two variables is of the form $v = u^{\frac{3}{5}}$, we may write this as $\log v = \frac{3}{5} \log u$. Here we construct the adjacent scales $x = m \log v$ and $x = m\ (\frac{3}{5} \log u)$, *i.e.*, two logarithmic scales with moduli m and $3\ m/5$ respectively. We use chart $3c$ and get Fig. $4d$, from

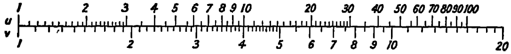


FIG. $4d$.

which we can read v when u lies within the limits 1 to 100, or read u when v lies within the limits 1 to 20.

We may similarly construct two adjacent scales for the relation $v = u^p$, where p is any positive number. The chart Fig. $3c$ may conveniently be used for this purpose. We may write the relation as $\log v = p \log u$, and we pick out on the chart the scales $x = m \log v$ and $x = (pm) \log u$, *i.e.*, with moduli m and pm. Since the axes carrying these scales are parallel with origins in the same perpendicular, any perpendicular to the axes will cut out corresponding values of u and v. If $p < 1$ we may use the primary logarithmic scale for the v scale. If $p > 1$, we write the relation in the form $u = v^{1/p}$ and proceed similarly.

If in the relation $v = u^p$, p is a negative number, say, $-q$, then $v = u^{-q}$. If we write $v = 10\ u^{-q}$, we merely shift the position of the decimal point in the value of v; then $\log v = \log 10 - q \log u$. Construct

the adjacent scales $x = m \log v$ and $x = m\,(\log 10 - q \log u) = m - (qm) \log u$ from the same origin; the latter scale is merely the scale $x = (qm) \log u$ constructed from the point $x = m$ as starting point and

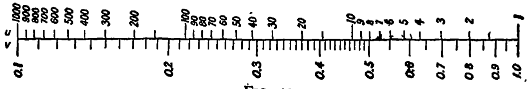


FIG. 4e.

proceeding to the left. Fig. 4e represents the relation $v = u^{-\frac{1}{3}}$ constructed with the help of chart 3c.

5. Sliding scales. — Consider two functions $f(u)$ and $F(v)$ and construct their scales $x = mf(u)$ and $x = mF(v)$. If these scales are placed adjacent with their origins coinciding or in the same perpendicular to the axes (we shall call this *the stationary position*), then for any pair of values u and v opposite each other, we have $OA = O'B$ (Fig. 5a), and hence, $mf(u) = mF(v)$, or

$$\textit{(in the stationary position)} \quad f(u) = F(v). \qquad \text{(I)}$$

This relation was illustrated in the examples of Art. 4.

If now one of the scales is slid along the other scale through *any* distance d, then for any pair of values of u and v opposite each other, we have $OA - O'B = d$ (Fig. 5b), or $mf(u) - mF(v) = d$, or $f(u) - F(v) =$

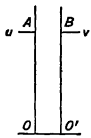


FIG. 5a.

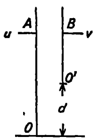


FIG. 5b.

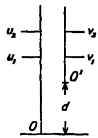


FIG. 5c.

$d/m = constant$, for d and m are independent of the pair of values of u and v considered; hence,

$$\textit{(after sliding)} \qquad f(u) - F(v) = constant. \qquad \text{(II)}$$

If u_1, v_1 and u_2, v_2 are two pairs of values of u, v opposite each other (Fig. 5c), then by (II), we may write

$$f(u_1) - F(v_1) = f(u_2) - F(v_2). \qquad \text{(III)}$$

Equations (I) and (II) are the important equations for the construction of stationary and sliding adjacent scales, illustrating the principles upon which the use of slide rules is founded.

As an example, consider the scales $x = m \log u$ and $x = m \log v$. If these are placed adjacent, then, in the stationary position, by (I), $\log u = \log v$ or $u = v$, and after sliding, by (II), $\log u - \log v = constant$, or $\log \frac{u}{v} = constant$, or $\frac{u}{v} = constant$ for any pair of values of u and v opposite each other, and $\frac{u_1}{v_1} = \frac{u_2}{v_2}$ for any two pairs of values of u and v opposite each other. Thus if any three of the last four quantities are given, the

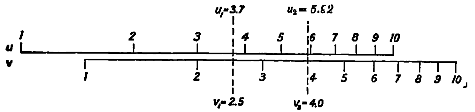


FIG. 5*d*.

fourth quantity may be found at once; thus if u_1, v_1, u_2 are given, slide the scales until v_1 is opposite u_1, and read v_2 opposite u_2. This is illustrated in Fig. 5*d*, where we read $\frac{3.7}{2.5} = \frac{5.92}{4.00}$.

We may perform the same operation by means of a single logarithmic scale $x = m \log u$ sliding along an unmarked axis (Fig. 5*e*).

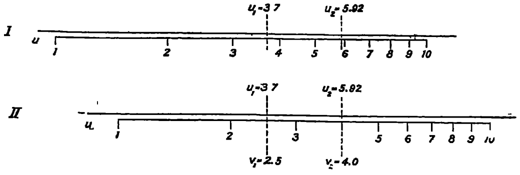


FIG. 5*e*.

1st position: place the scale $x = m \log u$ adjacent to the unmarked axis and mark on the latter the values u_1 and u_2.

2d position: slide the scale $x = m \log u$ until v_1 of this scale falls opposite u_1 of the unmarked axis; then read v_2 of the scale opposite u_2 of the unmarked axis.

It is evident that simple multiplication and division are special cases of the above, for if $u_1 = 1$ or 10, then $v_2 = u_2 \cdot v_1$ or $10\, u_2 \cdot v_1$, and if $v_2 = 1$ or 10, then $u_2 = u_1/v_1$ or $10\, u_1/v_1$.

6. The logarithmic slide rule.* —This instrument consists of several parallel logarithmic scales and one uniform scale, some on the stock of the rule and others on the slide. Any two of these scales may be placed adjacent by means of a glass runner which has a fine hair line scratched on its under side and which is adjusted so that the hair line is always perpendicular to the axes (Fig. 6).

All logarithmic slide rules do not carry the same number of scales. The following is a description of the scales and their equations on the

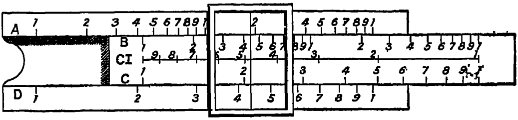


FIG. 6.

modern Mannheim standard rule (polyphase or duplex), commonly called "the 10-inch rule." The length of the graduated part of the rule is 25 cm. and we shall designate this length by m. The scales are distinguished by the letters $A, B, C, \ldots$. We shall use the corresponding small letters $a, b, c, \ldots$, to represent numbers on these scales.

$$L : x = ml. \qquad C : x = m \log c. \qquad D : x = m \log d.$$

$$A : x = \frac{m}{2} \log a. \qquad B : x = \frac{m}{2} \log b. \qquad CI : x = m \log \frac{10}{ci}.$$

$$K : x = \frac{m}{3} \log k. \qquad S : x = \frac{m}{2} \log (100 \sin s). \qquad T : x = m \log (10 \tan t).$$

The C and D scales are graduated so that we can easily read three figures in any part of these scales. Rules for the position of the decimal point may be given, but in computing it is best to disregard all decimal points and to estimate the position of the decimal point in the final result.

The following are some of the relations which arise through the application of the principles of stationary and sliding scales to this type of rule. (Other illustrations will be found in the manuals issued by the manufacturers.) We shall designate the stationary or initial position by (I)

* *Historical Note.* Gunter invented the logarithmic scale and used compasses to calculate with it (1620). Oughtred invented the straight logarithmic slide rule, consisting of two rulers each bearing a logarithmic scale, which were slid along each other by hand (1630). Rules in which the slide worked between parts of a fixed stock were known in England in 1654. Robertson constructed the first runner in 1775. Mannheim designed the modern standard slide rule (1850). Roget invented the log-log scale in 1815. See F. Cajori's History of the Logarithmic Slide Rule.

and the position after sliding by (II). Numbers opposite each other are designated by the same subscript.

(1) L and D:

(I) $l = \log d$ and $d = \text{antilog}\, l$. (l is only the mantissa.)

(2) C and D (or A and B):

(I) $\log c = \log d, \quad \therefore \quad c = d.$

(II) $\log c - \log d = const., \quad \therefore \quad \frac{c}{d} = const.$ or $\frac{c_1}{d_1} = \frac{c_2}{d_2}$.

Multiplication: $p \times q = y$ or $\frac{1}{p} = \frac{q}{y}$, $\therefore \quad \frac{C}{D} \left| \frac{1 \text{ or } 10}{\text{one factor}} \right| \frac{\text{other factor}}{\text{product}}$.

Division: $\frac{p}{q} = y$ or $\frac{q}{p} = \frac{1}{y}$, $\therefore \quad \frac{C}{D} \left| \frac{\text{divisor}}{\text{dividend}} \right| \frac{1 \text{ or } 10}{\text{quotient}}$.

(3) D and CI:

(I) $\log d = \log \frac{10}{ci}, \quad \therefore \quad d = \frac{10}{ci}$ and $ci = \frac{10}{d}$ (for finding reciprocals).

It is evident that multiplying or dividing d by c is equivalent to dividing or multiplying d by ci. If the rule does not contain a CI scale, we may invert the slide so that the C scale slides along the A scale, thus transforming the C scale into a CI scale.

(II) $\log d - \log \frac{10}{ci} = const. \quad \therefore \quad d \times ci = const.$ or $d_1 \times ci_1 = d_2 \times ci_2$.

(4) D and A (or C and B):

(I) $\log d = \frac{1}{2} \log a, \quad \therefore \quad d = \sqrt{a}$ and $a = d^2$.

To find $\sqrt{a}$, divide a, as in arithmetic, into groups of two figures beginning at the decimal point; the left-hand group may contain only one significant figure. Thus, the left-hand groups in 45'.60', .45'60', .00'45'6 are said to contain two figures, while the left-hand groups in 4'56., 4'.56', .04'56', .00'04'56' are said to contain only one significant figure. We read a in the first half or second half of the A scale according as the left-hand group contains one or two figures.

(5) D and B (or C and A):

(I) $\log d = \frac{1}{2} \log b, \quad \therefore \quad d = \sqrt{b}$ and $b = d^2$.

(II) $\log d - \frac{1}{2} \log b = const., \therefore \frac{d}{\sqrt{b}} = const., \therefore \frac{d_1}{\sqrt{b_1}} = \frac{d_2}{\sqrt{b_2}}$ and $\frac{d_1^2}{b_1} = \frac{d_2^2}{b_2}$.

(6) D and K:

(I) $\log d = \frac{1}{3} \log k, \quad \therefore \quad d = \sqrt[3]{k}$ and $k = d^3$.

To find $\sqrt[3]{k}$, divide k, as in arithmetic, into groups of three figures beginning at the decimal point; the left-hand group may contain only one or two significant figures. Thus, the left hand groups in 456.',

.456′, .000′456′ are said to contain three figures, the left-hand groups in 45′.6, .045′6, .000′045′6 are said to contain two figures, while the left-hand groups in 4′.56, .004′56 are said to contain one figure. We read k in the first, middle or last third of the K scale according as the left-hand group contains one, two or three figures.

(7) C and K:

(I) $\log c = \frac{1}{3}\log k, \quad \therefore \quad c = \sqrt[3]{k}$ and $k \doteq c^3$.

(II) $\log c - \frac{1}{3}\log k = const., \quad \therefore \quad \frac{c}{\sqrt[3]{k}} = const., \quad \therefore \quad \frac{c_1}{\sqrt[3]{k_1}} = \frac{c_2}{\sqrt[3]{k_2}}$.

(8) A and K (or B and K):

(I) $\frac{1}{2}\log a = \frac{1}{3}\log k, \quad \therefore \quad \sqrt{a} = \sqrt[3]{k}$, or $a = k^{\frac{2}{3}}$ and $k = a^{\frac{3}{2}}$.

(9) A and S:

(I) $\frac{1}{2}\log a = \frac{1}{2}\log(100 \sin s), \quad \therefore \quad a = 100 \sin s$ and $s = \sin^{-1}\frac{a}{100}$.

(II) $\frac{1}{2}\log a - \frac{1}{2}\log(100 \sin s) = const., \therefore \frac{a}{\sin s} = const.$ or $\frac{a_1}{\sin s_1} = \frac{a_2}{\sin s_2}$.

The last relation may be used in the solution of oblique triangles.

To find $y = a \sin s$, we set $\frac{a}{\sin 90^\circ} = \frac{y}{\sin s}$.

To find $y = \frac{a}{\sin s}$, we set $\frac{a}{\sin s} = \frac{y}{\sin 90^\circ}$.

We also note that $\cos s = \sin(90^\circ - s)$.

(10) D and T:

(I) $\log d = \log(10 \tan t), \quad \therefore \quad d = 10 \tan t$ and $t = \tan^{-1}\frac{d}{10}$.

(II) $\log d - \log(10 \tan t) = const., \quad \therefore \quad \frac{d}{\tan t} = const.$, or $\frac{d_1}{\tan t_1} = \frac{d_2}{\tan t_2}$.

To find $y = d \tan t$, set $\frac{d}{\tan 45^\circ} = \frac{y}{\tan t}$.

To find $y = \frac{d}{\tan t}$, set $\frac{d}{\tan t} = \frac{y}{\tan 45^\circ}$.

We also note that $\cot t = \tan(90^\circ - t)$.

7. The solution of algebraic equations on the logarithmic slide rule.— The relation between the D and CI scales expressed in Art. 6 (3), viz.: that, after sliding, the product of d and ci is the same for all such pairs of numbers opposite each other, may be used to assist in the solution of algebraic equations of the second and third degrees. Thus if we set $ci = 1$ over $d = q$, then over any number y on the D scale we shall find $\frac{q}{y}$ on the CI scale $\left(\text{since } 1 \times q = y \times \frac{q}{y}\right)$ and y^2 on the A scale; this is

illustrated in the accompanying diagram: $\begin{array}{c|c|c} A & y^2 & \\ \hline CI & \frac{q}{y} & 1 \\ \hline D & y & q \end{array}$ (Fig. 7). We may use $ci = 10$ instead of $ci = 1$ if necessary, but care must be taken in reading the position of the decimal point.

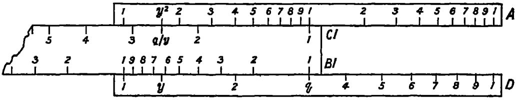


FIG 7.

(1) If we slide the runner until the readings on the D and CI scales are the same, then $y = \frac{q}{y}$ or $y^2 = q$ and $y = \pm\sqrt{q}$.

Thus, if $y^2 = 5$, or $y = \frac{5}{y}$, we have $\begin{array}{c|c|c} CI & 2.24 & 1 \\ \hline D & 2.24 & 5 \end{array}$, $\therefore\quad y = \pm 2.24$.

We also find $d = ci = 7.07$, but this is $\sqrt{50}$.

(2) If we slide the runner until the reading on the D scale plus p equals the reading on the CI scale, then $y + p = \frac{q}{y}$ or $y^2 + py = q$.

Thus, if $y^2 + 3\,y = 5$ or $y + 3 = \frac{5}{y}$, we have $\begin{array}{c|c|c} CI & 4.19 & 1 \\ \hline D & 1.19 & 5 \end{array}$, $\therefore$ $y = 1.19$.

Since the sum of the roots of the equation $y^2 + py = q$ is $-p$, the other root is -4.19.

A negative root may be found by replacing y by $-y_1$; thus the negative root of $y^2 + 3\,y = 5$ is a positive root of $y_1^2 - 3\,y_1 = 5$.

(3) If we slide the runner until the readings on the A and CI scales are the same, then $y^2 = \frac{q}{y}$ or $y^3 = q$ and $y = \sqrt[3]{q}$.

Thus if $y^3 = 5$ or $y^2 = \frac{5}{y}$, we have $\begin{array}{c|cc} A & 2.92 & \\ \hline CI & 2.92 & 1 \\ \hline D & 1.71 & 5 \end{array}$, $\therefore\quad y = 1.71$.

We also find $a = ci = 13.6$ opposite $d = 3.68$, but this is $\sqrt[3]{50}$.

We also find $a = ci = 63\,0$ opposite $d = 7.94$, but this is $\sqrt[3]{500}$.

(4) If we slide the runner until the reading on the A scale plus p equals the reading on the CI scale, then $y^2 + p = \frac{q}{y}$ or $y^3 + py = q$.

The nature of the roots of this cubic equation are determined as follows:

$$\frac{q^2}{4}+\frac{p^3}{27} \gtreqless 0 \begin{cases} \text{only 1 real root; if } q \text{ is } +\text{, root is } +\text{; if } q \text{ is } -\text{, root is } -. \\ \text{3 real roots of which 2 are equal.} \\ \text{3 real and unequal roots; 1 root is } + \text{ and 2 roots are } - \text{ or 1} \\ \quad \text{root is } - \text{ and 2 roots are } +. \end{cases}$$

To find the negative roots, we replace y by $-y_1$, and the positive roots of the resulting equation are the negative roots of the original equation.

We also note that the sum of the three roots of the equation $y^3 + py = q$ is *zero*. The complete cubic equation $z^3 + az^2 + bz + c = 0$ must first be reduced to the form $y^3 + py = q$ by the substitution $z = y - \frac{a}{3}$. To facilitate the comparison of the A and CI scales, it is well to invert the slide so that the C scale is transformed into a CI scale and slides along the A scale.

Thus, if $y^3 + 3y = 5$ or $y^2 + 3 = \frac{5}{y}$, there is only one positive root since $\frac{q^2}{4}+\frac{p^3}{27} = \frac{25}{4} + 1 > 0$ and q is positive. This positive root is found by the setting $\begin{array}{c|c|c} A & 1.33 & \\ \hline CI & 4.33 & 1 \\ \hline D & 1.153 & 5 \end{array}$, $\therefore \quad y = 1.153$.

Again, if $y^3 - 4y = 2$ or $y^2 - 4 = \frac{2}{y}$, there are three real roots, since $\frac{q^2}{4}+\frac{p^3}{27} = 1 - \frac{64}{27} < 0$. The positive root is found by the setting

$$\begin{array}{c|c|c} A & & 4.9 \\ \hline CI & 10 & 0.9 \\ \hline D & 2 & 2.21 \end{array}, \quad \therefore \quad y = 2.21.$$

To find a negative root, replace y by $-y_1$, and get $-y_1^3 + 4y_1 = 2$, or $y_1^2 - 4 = -\frac{2}{y_1}$, or $y_1^2 + \frac{2}{y_1} = 4$. We have the setting $\begin{array}{c|cc} A & 2.8 & \\ \hline CI & 1.2 & 1 \\ \hline D & 1.67 & 2 \end{array}$,

$\therefore \quad y_1 = 1.67$ and $y = -1.67$.

To find the third root, we note that the sum of the three roots is zero, $\therefore$ $2.21 - 1.67 + y = 0$, or $y = -0.54$. We may also find this root by the setting $\begin{array}{c|cc} A & & 0.29 \\ \hline CI & 10 & 3.71 \\ \hline D & 2 & 0.54 \end{array}$, $\therefore \quad y_1 = 0.54$ and $y = -0.54$.

8. The log-log slide rule. — Suppose we wish to construct a slide rule for finding any power (integer or fractional) of a number, *i.e.*, for finding the value of the expression $n_2 = n_1^{\frac{c_2}{c_1}}$. To find the equations of

the required scales we must write this equation in the form (II) or (III) of the principle expressed in Art. 5. Taking logarithms, we get $\log n_2 = \frac{c_2}{c_1} \log n_1$, or $\frac{\log n_1}{c_1} = \frac{\log n_2}{c_2}$, and taking logarithms again, we get $\log \log n_1 - \log c_1 = \log \log n_2 - \log c_2$, or $\log \log n - \log c = const.$ The equations of our scales are therefore $x = m \log \log n$ and $x = m \log c$. The initial point of the N scale would be marked $n = 10$ (since $x = m \log \log 10 = m \log 1 = 0$), and the end point would be marked $n = 10^{10}$ (since $x = m \log \log 10^{10} = m \log 10 = m$) if our scale is to be m cm. long. The range from $n = 10$ to $n = 10^{10}$ is not a convenient range for n, so that it has been found best to modify the equation of this scale somewhat. An instrument called the *log-log rule* (Fig. 8) has been constructed

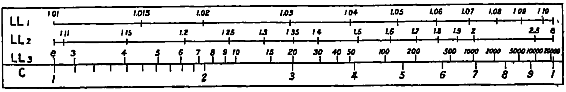


FIG. 8.

in which the equation of the n scale is $x = m \log (100 \ln ll)$ (where $\ln ll = \log_e ll$ and $e = 2.71828 \ldots$, the base of Napierian or natural logarithms). The scale is broken into three parallel scales of length $m = 25$ cm.:

the first, marked LL_1 with a range from $ll = e^{0.01}$ ($= 1.01$ approx.) to $ll = e^{0.1}$,

the second, marked LL_2 with a range from $ll = e^{0.1}$ to $ll = e$,

the third, marked LL_3 with a range from $ll = e$ to $ll = e^{10}$ ($= 22{,}000$ approx.).

By sliding the adjacent scales $x = m \log (100 \ln ll)$ and $x = m \log c$, we have $\log (100 \ln ll) - \log c = const.$, or $\frac{100 \ln ll}{c} = const.$, or $\sqrt[c]{ll} = const.$, or $\sqrt[c_1]{ll_1} = \sqrt[c_2]{ll_2}$, or $ll_2 = ll_1^{\frac{c_2}{c_1}}$; hence we have the setting $\left.\frac{LL}{C}\right| \frac{ll_2 \quad ll_1}{c_2 \quad c_1}$

If we set $c_1 = 1$ or 10 opposite ll_1, we have $ll_2 = ll_1^{c_2}$ or $ll_2 = ll_1^{\frac{c_2}{10}}$. Of course on the scale $x = m \log c$, c_2 and $c_2/10$ have the same position, but on the LL scales the decimal point must be left in its original position It is easy to see on which of the three LL scales the result is to be read;

thus $y = 2^{4.5}$ gives the setting $\begin{array}{c|ll} LL_2 & & 2 \\ \hline LL_3 & 22.6 & \\ \hline C & 4.5 & 1 \end{array}$, $\therefore \quad y = 22.6$

and $y = 2^{0.45}$ gives the setting $\begin{array}{c|ll} LL_2 & 1.366 & 2 \\ \hline LL_3 & & \\ \hline C & 0.45 & 1 \end{array}$, $\therefore \quad y = 1.366.$

We note above that the smallest value of ll is 1.01. Values of $ll \leqq 0.99$ may at once be replaced by their reciprocals, and the reciprocals of the final result taken, since $\frac{1}{ll_2} = \left(\frac{1}{ll_1}\right)^{c_2}$.

The LL and the C scales in their initial position may also be used to find directly the natural logarithm of a number, for we have $\log(100 \ln ll) = \log c$ or $\ln ll = \frac{c}{100}$.

It is evident that *compound interest* problems are very easily solved with the log-log rule. Thus, the amount, A, of the sum of \$1.00 placed at r per cent interest for n years and compounded q times a year, is given by $A = \left(1 + \frac{r}{100\,q}\right)^{nq}$; the required setting is then $\frac{LL}{C}\left|\frac{A}{nq}\right|\frac{\left(1 + \frac{r}{100\,q}\right)}{1 \text{ or } 10}$.

Many other illustrations may be found in the manuals published by the manufacturers.

9. Various other straight slide rules. — As another illustration of the use of sliding scales, let us construct a slide rule for the expression $\frac{1}{u} + \frac{1}{v}$. If we choose for our scales $x = m\left(\frac{1}{u}\right)$ and $x = m\left(10 - \frac{1}{v}\right)$, then for any position of our scales we shall have $\frac{1}{u} - \left(10 - \frac{1}{v}\right) = const.$ or $\frac{1}{u} + \frac{1}{v} = const.$, or $\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{u_2} + \frac{1}{v_2}$. If we choose the modulus, m, to be 1 in., and the total length of our scales to be 10 in., then the range

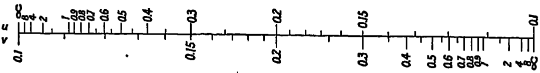
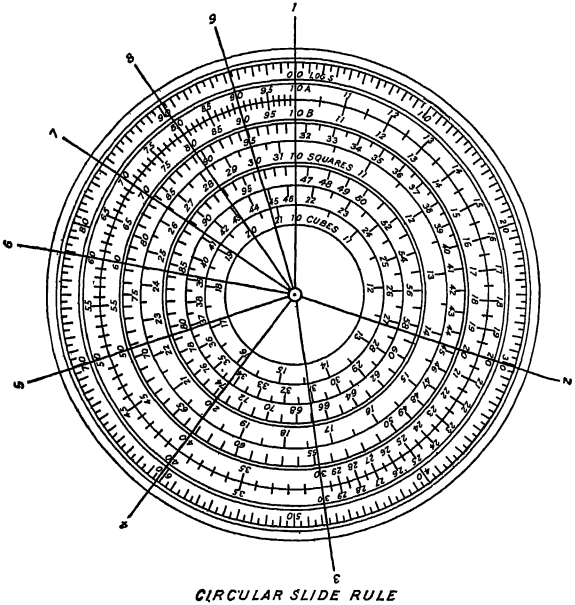


FIG. 9.

of u is from $u = \infty$ $(x = 0)$ to $u = 0.1$ $(x = 10)$, and the range of v is from $v = 0.1$ $(x = 0)$ to $v = \infty$ $(x = 10)$. (Fig. 9.) Now if we set $v_2 = \infty$ opposite $u_2 = u$, we shall have $\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{u}$, and we may read off any one of the three quantities u_1, v_1, u if the other two are known. This rule may be used to solve the equation $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$, where R is the combined electrical resistance of two parallel resistances R_1 and R_2, or to solve the equation $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$, where f is the principal focal distance of a lens and f_1 and f_2 are conjugate focal distances.

A large number of slide rules have been constructed for solving various special equations in engineering practice. Among these may be mentioned: stadia slide rules for measuring the horizontal distance and vertical height when the rod reading and the elevation of telescope are known; Nordell's sewer rule for solving Kutter's formula for circular sewers; Hudson's horse power computing scale for obtaining the indicated H.P. of an engine (this rule has two slides); Hazen-Williams hydraulic rule for finding the velocity of the flow of water through pipes (see chart on p. 61).



CIRCULAR SLIDE RULE

FIG. 10.

10. Curved slide rules. — Divide the angular magnitude about a point, viz., 2π radians, into 1000 equal parts by straight rays drawn through the point. Choose one of these rays as initial ray and mark it with the number 1 (for $0 = \log 1$); mark the ray at the end of 301 parts

with the number 2 (for 0.301 = log 2), and the ray at the end of 477 parts with the number 3 (for 0.477 = log 3), etc. Then the angle will be divided logarithmically (Fig. 10). The circumference of any circle drawn with the point as center will likewise be divided logarithmically, the points on the circumference carrying the same numbers as the rays through them.

Designating such a circumference by A, the numbers on it by a, and its radius by r_a inches, the equation of the scale on A is $x = (2\pi r_a) \log a$, *i.e.*, the point marked a_1 is at a distance of $(2\pi r_a) \log a_1$ inches from the initial point, $a = 1$, measured along the circumference. We now draw a concentric circumference, B, of radius r_b, carrying a scale $x = (2\pi r_b) \log b$ and so constructed that the plane of the B-circumference can rotate about the center. If in the initial position of the scales, *i.e.*, when the numbers $a = 1$ and $b = 1$ are on the same ray, a ray cuts out the numbers a_1 and b_1, then we have

$$\frac{(2\pi r_a) \log a_1}{(2\pi r_b) \log b_1} = \frac{r_a}{r_b}, \quad \therefore \quad \log a_1 = \log b_1 \text{ and } a_1 = b_1.$$

If, after rotation of the B-scale through any angle, two rays cut the scales in a_1; b_1 and a_2, b_2, then

$$\frac{(2\pi r_a) \log a_1 - (2\pi r_a) \log a_2}{(2\pi r_b) \log b_1 - (2\pi r_b) \log b_2} = \frac{r_a}{r_b}, \quad \therefore \quad \log a_1 - \log a_2 = \log b_1 - \log b_2.$$

$$\therefore \quad \frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

Hence the ratio of two numbers on the same ray is constant. This principle of rotating circular scales is therefore similar to the principle of sliding straight scales. Thus for

Multiplication: $y = a \cdot b$ or $\frac{a}{1} = \frac{y}{b}$, we set $\frac{A}{B} \left| \frac{\text{one factor}}{1} \right| \frac{\text{product}}{\text{other factor}}$.

Division: $y = \frac{a}{b}$ or $\frac{a}{b} = \frac{y}{1}$, we set $\frac{A}{B} \left| \frac{\text{dividend}}{\text{divisor}} \right| \frac{\text{quotient}}{1}$.

One advantage of such a circular rule lies in the fact that we avoid the difficulty of running off the rule, as often happens in setting with the straight slide rule.

In the instrument called "Sexton's Omnimetre," which Fig. 10 reproduces in part, the A- and B-circles have the same radius, about 3 inches, so that it is approximately equivalent to a straight rule of 18 inches. A ray drawn on a strip of celluloid capable of revolving about the center aids in the setting of the scales. The plane of the B-circle also contains the scale $x = (4\pi r_c) \log c$; and for two numbers, b and c, on the same ray, we have, in the initial position,

$$\frac{(4\pi r_c) \log c}{(2\pi r_b) \log b} = \frac{r_c}{r_b}, \quad \text{or} \quad \frac{2 \log c}{\log b} = 1, \quad \therefore \quad b = c^2 \text{ and } c = \sqrt{b}.$$

It is evident that if c is to vary from 1 to 10, the C scale must consist of two concentric circumferences, on one of which c varies from $c = 1$ (or $x = 0$) to $c = \sqrt{10}$ (or $x = 2\pi r_c$) and on the other from $c = \sqrt{10}$ to $c = 10$ (or $x = 4\pi r_c$). The C and B scales thus serve for finding squares and square roots. We may also combine the C and A scales, and after rotation we have $a_1/c_1^2 = a_2/c_2^2$. The instrument also contains three concentric circumferences for the scale $x = (6\pi r_k) \log k$; and a combination with the B-scale, in the initial position, gives $b = k^3$ or $k = \sqrt[3]{b}$. The instrument further contains scales for sines, tangents, and versines, and a scale of equal parts.

There are other forms of curved rules. "Lilly's Improved Spiral Rule," a disk 13 inches in diameter, consists of a spiral logarithmic scale and a circular scale of equal parts, and is equivalent to a straight rule of about 30 feet long; it gives results correct to 4 figures. "Thacher's Rule" consists of two logarithmic scales one on a cylinder and the other on a set of 20 parallel bars external to the cylinder. This is really an amplification of the straight slide rule, involving the same principle in its use; the rule gives four figures correctly and a fifth may be estimated.*

EXERCISES.

(*Note.* For the constructions in Exs. 4–10 use charts of uniform and logarithmic scales, Art. 3)

1 Construct scales for the function $\sqrt{u}$ ($u = 0$ to $u = 100$) if m is 1 in., 0.5 in., 0.2 in., respectively.

2. Construct scales for the function $\log u$ ($u = 1$ to $u = 10$) if $m = 5$ in., 10 in., $\frac{10}{3}$ in., respectively.

3. Construct a scale for the function $\log \frac{10}{u}$ if $m = 10$ in.

4. Construct adjacent scales for converting inches (I) to centimeters (C); we have $C = 2.54\, I$.

5. Construct adjacent scales for converting cu. ft. per sec. (C) to million gallons per hour (G); we have $G = 0.0269\, C$.

6. Construct three adjacent scales for converting foot-lbs. per sec. (F) into horsepower ($H.P.$) and kilowatts (K); we have $H.P. = 1.818 \times 10^{-3} F$, and $K = 1.356 \times 10^{-3} F$.

7. Construct adjacent logarithmic scales for the following:

$$(a)\ v = \frac{1}{u}, \qquad (b)\ v = u^2; \qquad (c)\ v = \sqrt[3]{u};$$

$$(d)\ v = \frac{1}{u^3}; \qquad (e)\ v = u^{-\frac{1}{2}}; \qquad (f)\ v = u^{\frac{3}{2}}.$$

8. Construct adjacent logarithmic scales for $A = \frac{\pi D^2}{4 \times 144}$, where D = diameter in inches and A = area of circle in sq. ft.

* For descriptions and illustrations of this rule and other rules, see "Methods of Calculation, a Handbook of the Exhibition at the Napier Tercentenary Celebration," published by G. Bell & Sons, London.

9. Construct adjacent logarithmic scales for $h = \frac{144\,P}{62.5}$, where h = pressure head in ft. and P = pressure in lbs. per sq. in. for the flow of water.

10. Construct adjacent logarithmic scales for $h = \frac{v^2}{2\,g} = \frac{v^2}{64.4}$, where h = velocity head in ft. and v = velocity in ft. per sec. for the flow of water.

11. Show how to construct a slide rule for the relation $V^2 - v^2 = k\,(\cos T - \cos t)$.

12. Solve by means of the logarithmic slide rule the following equations:

(a) $y^3 + 3\,y - 7 = 0$; (b) $y^3 + y + 5 = 0$;
(c) $y^3 - y^2 - 6\,y + 1 = 0$; (d) $y^3 + y^2 + y - 1 = 0$;
(e) $y^3 - 3\,y^2 + 1 = 0$.

CHAPTER II.

NETWORK OF SCALES. CHARTS FOR EQUATIONS IN TWO AND THREE VARIABLES.

11. **Representation of a relation between two variables by means of perpendicular scales.** — A relation $\phi(u, v) = 0$ between two variables u and v may be represented by means of two perpendicular scales instead of two adjacent scales. Construct the scales $x = m_1 f(u)$ and $y = m_2 F(v)$ where $f(u)$ and $F(v)$ are any functions of u and v, on two perpendicular axes OX and OY, and through the points marked on these scales draw perpendiculars to the axes (Fig. 11*a*). Any pair of values of u and v that satisfy the equation $\phi(u, v) = 0$ will determine a point, viz., the intersection of the corresponding perpendiculars to the axes; thus the pair of values u_1, v_1 will correspond to the point of intersection of the perpendicular to OX through $u = u_1$ and the perpendicular to OY through $v = v_1$. The locus of all such points is a curve which is said to represent the relation $\phi(u, v) = 0$. The rectangular or Cartesian equation of this curve referred to the axes OX and OY may be found by solving the equations of the scales, $x = m_1 f(u)$ and $y = m_2 F(v)$, for u and v in terms of x and y, and substituting these values in $\phi(u, v) = 0$.

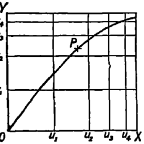


FIG. 11*a*.

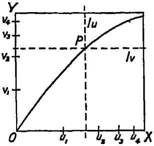


FIG 11*b*.

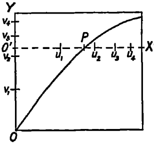


FIG. 11*c*.

It is evident that the nature of the locus by which the relation $\phi(u, v) = 0$ is represented, varies with the equations of the scales. *If possible, it is well to choose the scales so that the Cartesian equation is of the first degree in x and y, for then the representing curve will be a straight line.*

Having drawn the representing curve, and given a value of u, say u_k, we can find the corresponding value of v, say v_k, in one of three ways:

Fig. 11*a*. Here a network of perpendiculars to the axes is already drawn. Follow the perpendicular through u_k on OX until it cuts the

curve in the point P, and read v_k at the foot of the perpendicular from P to OY.

Fig. 11*b*. On a transparent sheet, draw two perpendicular index lines, I_u and I_v, intersecting in a point P. Slide the sheet so that the point P moves along the curve, keeping the index lines parallel to the axes; then, when I_u cuts OX in u_k, I_v will cut OY in v_k. (It is a simple mechanical matter to keep the index lines parallel to the axes)

Fig. 11*c*. Draw the scale $x = m_1 f(u)$ with axis $O'X'$ on a transparent strip. Slide the strip perpendicular to OY until the point u_k falls on the curve; then at O' read v_k.

In any case, the interpolation of u_k and v_k on the u and v scales is easily done by sight.

12. Some illustrations of perpendicular scales. —

(1) Consider the relation $v = u^2$. If we construct two uniform scales $x = mu$ and $y = mv$ on OX and OY respectively, and draw perpendiculars to the axes through the points marked on the scales, we shall have the rulings of an ordinary piece of *rectangular coordinate paper*. (Fig. 12*a*.) Here, $v = u^2$ will be represented by the locus whose Cartesian equation is $y/m = x^2/m^2$ or $x^2 = my$, a parabola We plot this curve from a table of values of u and v satisfying the equation $v = u^2$. Note that we could have constructed the scales $x = m_1 u$ and $y = m_2 v$ with different moduli m_1 and m_2, but the corresponding Cartesian equation $m_2 x^2 = m_1^2 y$ still represents a parabola.

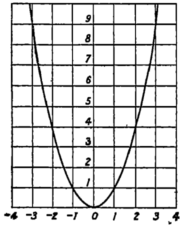


FIG. 12*a*.

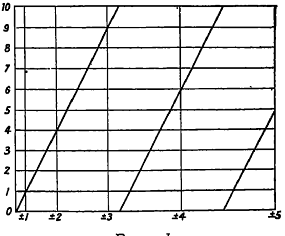


FIG. 12*b*.

(2) Again, we can represent the relation $v = u^2$ as follows: If we construct the scales $x = m_1 u^2$ and $y = m_2 v$ on OX and OY respectively, we shall have the rulings as in Fig. 12*b*, and our relation will be represented by the locus whose Cartesian equation is $y/m_2 = x/m_1$ or $y = m_2 x/m_1$, a straight line of slope m_2/m_1. In Fig. 12*b*, $m_1 = 0.1$ in. and $m_2 = 0.2$ in. The line may be plotted by means of two sets of values of u and v which satisfy $v = u^2$, such as $u = 0$, $v = 0$ and $u = 2$, $v = 4$, or by means of

one such set and the slope, 2, of the line. Note that the points on the scale $x = m_1u^2$ are marked with + and − values of u.

It is evident that the representation in Fig. 12*b* is much simpler than that in Fig. 12*a*. In the former, the straight line is much more easily constructed and every point on it is definitely determined, while, in the latter, the curve between plotted points is only approximated. Of course, it is easier to interpolate on the uniform u-scale in Fig. 12*a* than on the non-uniform u-scale in Fig. 12*b*. Furthermore, in Fig. 12*b*, we can project the point in which the representing line cuts $v = 10$ vertically on $v = 0$ and thus draw a second section of the line parallel to the first sec-

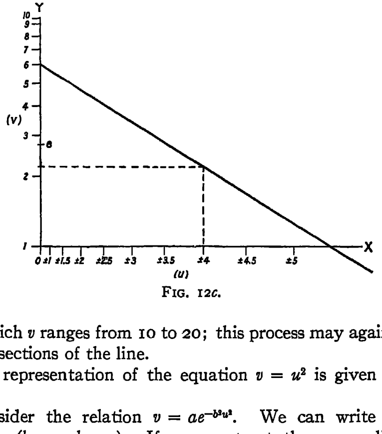


FIG. 12*c*.

tion, for which v ranges from 10 to 20; this process may again be used to get further sections of the line.

A third representation of the equation $v = u^2$ is given in the next article.

(3) Consider the relation $v = ae^{-b^2u^2}$. We can write this $\ln v = -b^2u^2 + \ln a$ ($\ln v = \log_e v$). If we construct the perpendicular scales $x = m_1u^2$ and $y = m_2 \ln v$, our relation will be represented by the straight line whose equation is $\frac{y}{m_2} = -\frac{b^2x}{m_1} + \ln a$. This line is easily constructed by means of the points $u = 0$, $v = a$ and $u = \frac{1}{b}$, $v = \frac{a}{e}$. In Fig. 12*c*, we have taken $a = 6$, $b = \frac{1}{4}$, $m_1 = 0.2$ in., $m_2 = 2$ in. A table of natural logarithms was used to construct the scale on OY.

13. **Logarithmic coördinate paper.** — Consider the relation $u^pv^q = a$, where p, q, and a are any numbers. We can write this $p \log u + q \log v = \log a$. If we construct the perpendicular scales $x = m \log u$ and $y = m \log v$ and draw the perpendiculars to the axes, we shall

have the rulings of a sheet of *logarithmic coordinate paper*. In Fig. 13, $m = 25$ cm. and u and v vary from 1 to 10. (Logarithmic paper can be constructed for larger ranges of the variables and with various moduli.) Our relation will be represented by the straight line whose Cartesian equation is $\frac{p}{m}x + \frac{q}{m}y = \log a$, which may be plotted by means of two pairs of corresponding values of u and v or by means of one pair

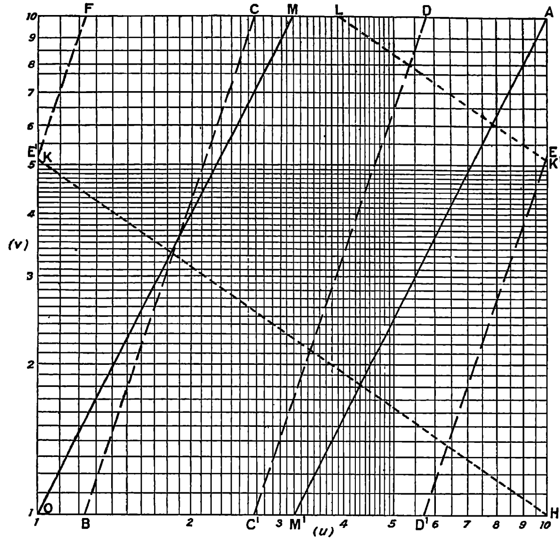


FIG. 13.

of values and the slope $-p/q$. A chart can thus be built up for a large number of equations in two variables. The following are some examples.

(1) For the relation $v = u^2$, the representing straight line passes through the points $u = 1, v = 1$ and $u = 3, v = 9$; this gives the section OM in Fig. 13. But since the scale on OY can also serve to represent $\log v$ for v from 10 to 100, we shall get another section, $M'A$, which can be drawn through $u = 4, v = 16$ and $u = 9, v = 81$, or through M' on OX vertically below M and parallel to OM. The two sections, OM and

$M'A$, will then represent the relation $v = u^2$, where u varies from 1 to 10 and v from 1 to 100, and hence for all values of u and v since the scale $x = m \log u$, for example, where u varies from 10^p to 10^{p+1} (p = integer) can be made to coincide with the scale $x = m \log u$, where u varies from 1 to 10. The position of the decimal point must be determined independently in each case. In finding u when v is given, divide v into groups of two figures each beginning at the decimal point, as in arithmetic (see Art. **6** (4)); if the left-hand group contains only one significant figure, use the section OM, if it contains two significant figures, use the section $M'A$; thus when $v = 0.64$, read $u = 0.8$, but when $v = 0.064$, read $u = 0.253$.

(2) For the relation $v = \frac{1}{6}\pi u^3$, the volume of a sphere in terms of its diameter, the representing straight line passes through the point $u = 2$, $v = \frac{4}{3}\pi$ and has a slope equal to 3; this gives the section BC in Fig. 13. We continue this line by projecting C into C' on OX and drawing $C'D$ parallel to $B'C$, then projecting D into D' on OX and drawing $D'E$ parallel to $C'D$, and complete this last section by projecting E into E' on OY and drawing $E'F$ parallel to $D'E$; F will project into the initial point B on OX. Our relation is completely represented by these sections for all values u and v. In finding u when v is given, divide v into groups of three figures each beginning at the decimal point as in arithmetic (see Art. **6** (6)); according as the left-hand group contains one, two, or three significant figures, use the first, second, or third section, respectively.

(3) For the relation $u \cdot v^{1.41} = 10$, where u is the pressure and v is the volume of a perfect gas, our first representing section, HK, passes through the point $u = 10$, $v = 1$ with slope $-1/1.41$; the second section, $K'L$, is easily constructed and these two sections will serve for the variation of v from 1 to 10. If the sections are continued, later sections will overlap the preceding ones.

14. **Semilogarithmic coördinate paper.** — Consider the relation $v = p \cdot q^u$, where p and q are any numbers. We can write this $\log v = u \log q + \log p$. If we construct the perpendicular scales $x = m_1 u$ and $y = m_2 \log v$, and draw the perpendiculars to the axes, we shall have the rulings of a sheet of *semilogarithmic coordinate paper*. In Fig. 14, $m_1 = m_2 = 25$ cm. and u varies from 0 to 1 while v varies from 0.1 to 1. (Semilogarithmic paper can be constructed for larger ranges of the variables and with various moduli.) Our relation will be represented by the straight line whose Cartesian equation is $\frac{y}{m_2} = \frac{x}{m_1} \log q + \log p$, which can be plotted by means of two pairs of corresponding values of u and v, or by means of one pair and the slope $m_2 \log q / m_1$. The following examples will serve to illustrate the use of semilogarithmic paper.

(1) The relation $v = 0.1\, e^{2.1u}$ can be written $\log v = (2.1 \log e)\, u + \log 0.1$, where e is the base of natural logarithms, and is represented

by the straight line (section OA in Fig. 14), which passes through the points $u = 0$, $v = 0.1$ and $u = 1$, $v = 0.817$. To extend the range of our variables we need not extend the chart but merely project A horizontally into A' on OY, draw $A'F'$ parallel to OA, project F' vertically into F'' on OX, and draw $F''E''$ parallel to OA; then for OA, u varies from 0 to 1 and v from 0.1 to 0.817, while for $A'F'$ and $F''E''$, u varies from 1 to 2 and v from 0.817 to 6.668.

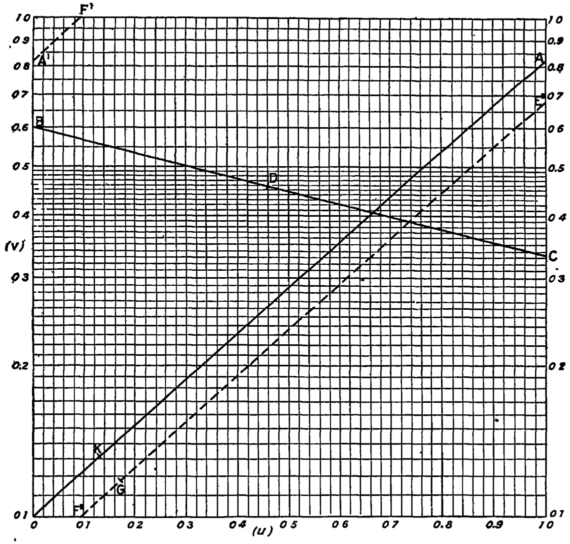


FIG. 14.

(2) Suppose we wish to find the values of u and v satisfying simultaneously the equations $v = 0.1\,e^{2.1u}$ and $v = 0.6\,e^{-0.591u}$. We represent each of these equations by a straight line (Fig. 14). The line representing the second equation passes through the points $u = 0$, $v = 0.6$ and $u = 1$, $v = 0.332$. At the point of intersection of these lines we read the required values $u = 0.663$, $v = 0.404$.

(3) To solve the equation $v = p \cdot q^v$ or $\log v = v \log q + \log p$ for the unknown quantity v, we draw the straight line representing the equa-

tion $v = p \cdot q^u$, and run our eyes along this line until we find the point where u and v are equal; this is the required value of v.

Thus to find the solution of $v = 0.6\, e^{-0.591 v}$, we draw the line BC representing the equation $v = 0.6\, e^{-0.591 u}$ and run our eyes along this line (watching the u and v scales) to the point D where we read $u = v = 0.46$ (Fig. 14).

Again, to find the solution of the equation $\log v = 0.912\, v - 1$, we draw the line OA representing the equation $\log v = 0.912\, u - 1$ [this equation is equivalent to the equation considered in (1), since $2.1 \log e = 0.912$ and $\log 0.1 = -1$] and run our eyes along this line until we read $u = v = 0.132$. We can find another value of v satisfying the equation by running our eyes along the section $F''E''$ until we read $u = v = 1.17$.

15. Rectangular coördinate paper — the solution of algebraic equations of the 2d, 3d, and 4th degrees. — We may use the rulings of a sheet of rectangular coördinate paper to solve graphically algebraic equations of the 4th, 3d, and 2d degrees. Let the scales be $x = u$ and $y = v$ where the modulus is 1.

(1) If we draw the parabola $y^2 = 2\,x$ and the circle $(x-h)^2 + (y-k)^2 = r^2$ with center at (h, k) and radius r, the ordinates of their points of intersection are found algebraically by eliminating x between these two equations and solving the resulting equation for y. From the first equation we have $x = y^2/2$, and substituting this in the second equation we get $\left(\frac{y^2}{2} - h\right)^2 + (y-k)^2 = r^2$, or

$$y^4 + 4\,(1-h)\,y^2 - 8\,ky + 4\,(h^2 + k^2 - r^2) = 0.$$

If we divide this last equation by t^4, where t is an arbitrary number, we get

$$\left(\frac{y}{t}\right)^4 + \frac{4\,(1-h)}{t^2}\left(\frac{y}{t}\right)^2 - \frac{8\,k}{t^3}\left(\frac{y}{t}\right) + \frac{4\,(h^2+k^2-r^2)}{t^4} = 0,$$

or

$$z^4 + az^2 + bz + c = 0,$$

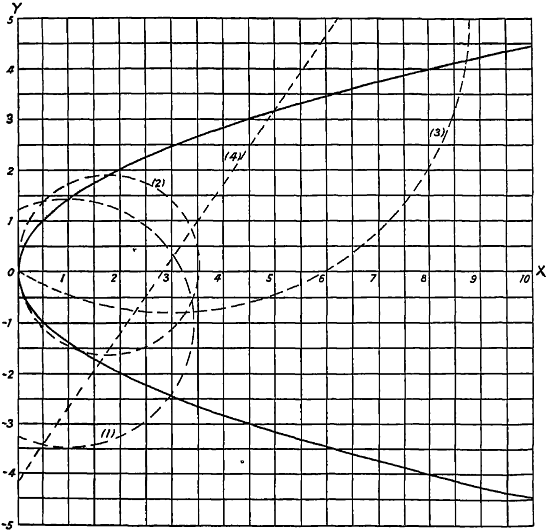
where $$z = \frac{y}{t},\quad a = \frac{4\,(1-h)}{t^2},\quad b = -\frac{8\,k}{t^3},\quad c = \frac{4\,(h^2+k^2-r^2)}{t^4}.$$

Conversely, the real roots of the equation $z^4 + az^2 + bz + c = 0$ are found by measuring the ordinates of the points of intersection of the parabola $y^2 = 2\,x$ and the circle with center at

$$h = \frac{4 - at^2}{4},\quad k = -\frac{bt^3}{8} \text{ and radius } r = \sqrt{h^2 + k^2 - \frac{ct^4}{4}},$$

where t is an arbitrary number, and dividing these ordinates by t. The introduction of t allows us to throw the center of the circle to a convenient point — always to the right of OY, or as near to or as far

from the vertex of the parabola as is convenient — such that the circle will not cut the parabola at an angle too acute for accurate reading of the ordinates. Note that the one parabola, $y^2 = 2x$, will serve for finding the real roots of all equations of the fourth degree (Fig. 15). To solve the complete equation $v^4 + pv^3 + qv^2 + rv + s = 0$, we first substi-



ANALYTIC CHART FOR SOLUTION OF ALGEBRAIC EQUATIONS

FIG. 15.

tute $v = z - p/4$ and this equation takes the form $z^4 + az^2 + bz + c = 0$, and then proceed as above.

Example 1. Let us find the real roots of the equation $z^4 + z - 1 = 0$. Here $a = 0$, $b = 1$, $c = -1$. Hence $h = 1$ and $k = -\frac{1}{8}t^3$. If we choose $t = 2$, the center is the point $(1, -1)$ and the radius is $\sqrt{6} = 2.45$. The circle (Fig. 15) cuts the parabola in two points whose ordinates are approximately $y = 1.4$ and $y = -2.4$. Hence $z = y/t = 0.7$ and $-1\ 2$.

(2) If $c = 0$ in the equation $z^4 + az^2 + bz + c = 0$, this equation becomes $z^4 + az^2 + bz = 0$ or $z(z^3 + az + b) = 0$ or $z = 0$ and $z^3 + az + b = 0$. One of the roots being zero, the circle will pass through the

origin or vertex of the parabola, and the other points of intersection will give the real roots of the cubic $z^3 + az + b = 0$. Hence, the real roots of the equation $z^3 + az + b = 0$ are found by measuring the ordinates of the points of intersection of the parabola $y^2 = 2x$ and the circle with center at $h = \frac{4 - at^2}{4}$, $k = -\frac{bt^3}{8}$ (where t is an arbitrary number) and passing through the vertex of the parabola, and dividing these ordinates by t. Note that the one parabola $y^2 = 2x$ will serve for finding the real roots of all fourth and third degree equations (Fig. 15) To solve the complete equation $v^3 + pv^2 + qv + r = 0$, we first substitute $v = z - \frac{p}{3}$ and this equation takes the form $z^3 + az + b = 0$, and then proceed as above.

Example 2. Let us find the real roots of the equation $z^3 - 3z - 1 = 0$. Here the center of the circle is at $h = \frac{4 + 3t^2}{4} = \frac{7}{4}$, $k = \frac{t^3}{8} = \frac{1}{8}$ if $t = 1$. We read $z = y = 1.88, -1.53, -0.35$ (Fig 15).

Example 3. Let us find the real roots of the equation $v^3 - 3v^2 + v - 4 = 0$. Let $z = v + 1$; then the equation becomes $z^3 - 2z - 5 = 0$. Here the center of the circle (Fig. 15) is at $h = 1 + \frac{1}{2}t^2 = 3$, $k = \frac{5}{8}t^3 = 5$, if $t = 2$. We read $y = 4.18$; hence $z = y/t = 2.09$, and $v = z - 1 = 1.09$.

(3) If we draw the parabola $y^2 = 2x$ and the straight line $y = mx + k$ of slope m and y-intercept k, the ordinates of their points of intersection are found from the equation $y^2 - \frac{2}{m}y + \frac{2k}{m} = 0$ If we divide this by t^2 (where t is an arbitrary number), we get $\left(\frac{y}{t}\right)^2 - \frac{2}{mt}\left(\frac{y}{t}\right) + \frac{2k}{mt^2} = 0$ or $z^2 + az + b = 0$, where $z = \frac{y}{t}$, $a = -\frac{2}{mt}$, $b = \frac{2k}{mt^2}$. Conversely, the real roots of the equation $z^2 + az + b = 0$ are found by measuring the ordinates of the points of intersection of the parabola $y^2 = 2x$ and the straight line of slope $m = -\frac{2}{at}$ and y-intercept $k = -\frac{bt}{a}$ (where t is an arbitrary number), and dividing these ordinates by t. Note that the one parabola $y^2 = 2x$ will serve for finding the real roots of all fourth, third, and second degree equations (Fig 15).

Example 4 Let us find the real roots of the equation $z^2 - 1.45z - 5.6 = 0$. Here the slope of the line is $m = \frac{2}{1.45t} = \frac{2}{1.45}$, and its y-intercept is $k = -\frac{5.6}{1.45t^2} = -3.86$, if $t = 1$. We read $z = y = -1.75$ and 3.20.

16. Representation of a relation between three variables by means of perpendicular scales. — An equation in three variables of the form

$\phi(u, v, w) = 0$ can be represented graphically by generalizing the method employed in Art. 11 for the representation of an equation in two variables. Thus, if we assign to w a value, say w_1, we shall have $\phi(u, v, w_1) = 0$, an equation in two variables u and v, which can be represented by the method of perpendicular scales, $x = m_1 f(u)$, $y = m_2 F(v)$, as a curve; this curve is marked with the number w_1. By assigning to w a succession of values, $w_1, w_2, w_3, \ldots$, we get a series of representing curves, each marked with its corresponding value of w. The equation in three variables is said to be represented by this network of curves. It is evident that the same equation $\phi(u, v, w) = 0$ can similarly be represented by a network of curves found by assigning a succession of values to u (or v) and marking each curve with its corresponding value of u (or v).

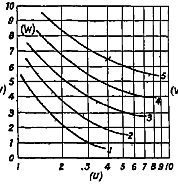


FIG 16a.

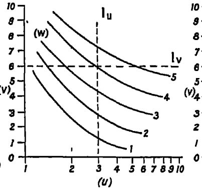


FIG. 16b.

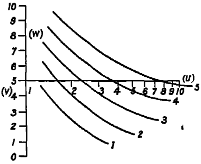


FIG. 16c.

Fig. 16a illustrates this representation. Given values of u and v, say u_k and v_k, we find the point (u_k, v_k) as the point of intersection of the corresponding vertical and horizontal lines, and read w_k from the curve passing through this point. If the point (u_k, v_k) falls between two of the curves w_j and w_l, we interpolate by sight the required value of w_k between w_j and w_l. Again, given values of u and w, say u_k and w_k, we find the point of intersection of the vertical u_k and the curve w_k, and read v_k from the horizontal passing through this point. Thus, in Fig. 16a, $u = 3$, $v = 4$ give $w = 3$; $u = 3$, $v = 4.5$ give $w = 3\ 3$; $u = 4$, $w = 5$ give $v = 6.6$.

As in Art. 11, we may avoid drawing the horizontals and verticals, and use a transparent sheet containing two perpendicular index lines, I_u and I_v (Fig. 16b); thus, if $u = 3$ and $v = 6$, slide the sheet keeping the index lines parallel to the axes until I_u passes through $u = 3$ and I_v passes through $v = 6$, and from the w-curve passing through their point of intersection read $w = 4$. If $u = 4$ and $w = 3.5$, slide the sheet keeping I_u perpendicular to OX until I_u passes through $u = 3$ and the point of intersection of the index lines lies on the curve $w = 3.5$, then I_v will cut OY in $v = 4.5$. Instead of the two index lines, we may also use a transparent strip carrying the u- (or v-) scale (Fig. 16c) and which slides perpendicular to the v- (or u-) scale; thus, if $v = 5$ and $w = 4$, we slide this strip

until OX passes through $v = 5$, and at its intersection with the curve $w = 4$, we read $u = 3.7$.

The task of drawing the w-curves is often very great, and it is therefore best, whenever convenient, to choose the scales for u and v so that the representing w-curves are straight lines. This will not only lessen the labor of construction but will evidently increase the accuracy of our charts.

17. Charts for multiplication and division. — This example will illustrate how the choice of scales determines the nature of the representing curves.

(1) The equation $uv = w$ can be represented by taking $x = mu$ and $y = mv$ for our two perpendicular scales, and drawing the corresponding

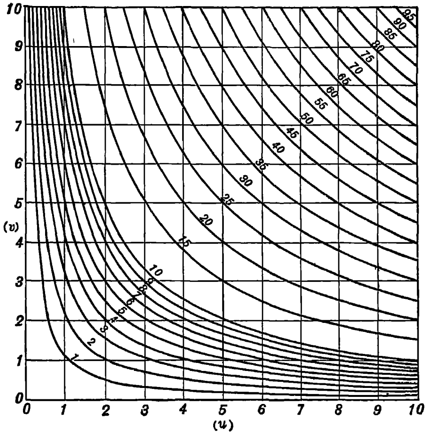


FIG 17*a*.

network. The equations of our representing w-curves are of the form $xy = m^2w$, a set of *rectangular hyperbolas* (Fig. 17*a*) which are quite difficult to draw and for which the interpolation is very inaccurate.

(2) The equation $uv = w$ can be represented by choosing $x = mu$ and $y = mw/10$ for our two perpendicular scales. The equations of our representing v-curves are of the form $y = vx/10$, a set of *radiating straight lines* of slope $v/10$. Fig. 17*b* illustrates this chart, where u and v vary from 1 to 10. The values of v are placed at the end of the representing lines. For saving of space, the values of w are placed at the points

where the horizontals cut the line $v = 10$ and the rulings above this diagonal need not be drawn. Of course, the position of the decimal point in the value of a variable may be changed with a corresponding change in the result. The great disadvantage of this chart is that the v-lines

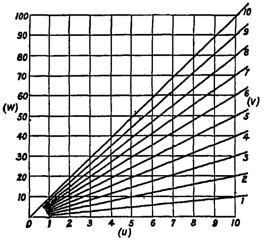


FIG. 17*b*.

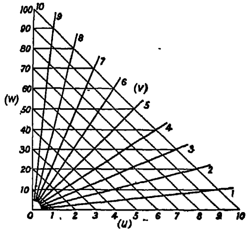


FIG. 17*c*.

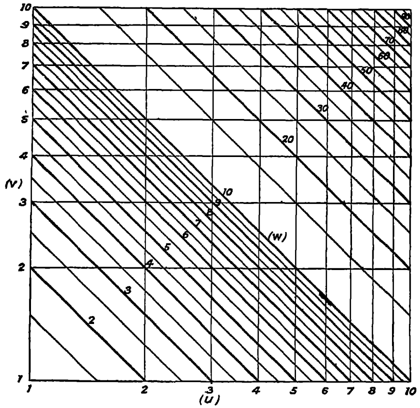


FIG. 17*d*.

converge to a point and also cut the horizontals at very small angles, thus making the reading quite inaccurate in parts of the chart. This may be remedied somewhat by rotating the verticals of the scale $x = mu$ through an angle of 45°, without changing the nature of the chart. This is illustrated in Fig. 17*c*. In Figs. 17*b* and 17*c*, the scale for u can be taken

over a range from 0 to 100 or, in general, from 0 to 10^k, similarly for the range of v, with corresponding change in the range for w.

(3) The equation $uv = w$ can be represented by writing it in the form $\log u + \log v = \log w$ and choosing $x = m \log u$, $y = m \log v$ for our two perpendicular scales, *i.e.*, we have the rulings of a sheet of logarithmic paper. The equations of our representing w-curves are of the form $x + y = m \log w$, a set of *parallel straight lines* (Fig 17*d*). These parallel lines are very easily drawn, for the line $w = k$, for example, cuts $u = 1$ (or the y-axis) at $v = k$, and cuts $v = 1$ (or the x-axis) at $u = k$; hence the line $w = k$ is a line joining the point on OX marked $u = k$ with the point on OY marked $v = k$. The ranges for u and v may be read from 10^p to 10^{p+1} with corresponding readings in the range for w.

The methods illustrated in (2) and (3) may be extended to any equation of the form $f(u) \cdot F(v) = \phi(w)$. If we choose $x = m_1 f(u)$ and $y = m_3 \phi(w)$ for our perpendicular scales, then the equations of our representing v-curves have the form $y = \frac{m_3}{m_1} F(v)\, x$, a set of *radiating* lines. But if we choose $x = m_1 \log f(u)$ and $y = m_2 \log F(v)$ for our perpendicular scales, then the equations of our representing w-curves have the form $\frac{x}{m_1} + \frac{y}{m_2} = \log \phi(w)$, a set of *parallel* lines.

18. Three-variable charts. Representing curves are straight lines. — The following examples illustrate the construction of charts for equations in three variables, where the perpendicular scales are so chosen that the representing curves are straight lines:

(1) The equation $w = \sqrt[5]{\alpha\beta^4}$ can be written $5 \log w = \log \alpha + 4 \log \beta$, and if we choose $x = m_1 \log \alpha$, $y = m_2 \log \beta$ for our perpendicular scales and let $m_1 = m_2 = 10$, the equations of our representing w-curves are of the form $x + 4y = 50 \log w$, a set of parallel straight lines. These lines have the slope $-\frac{1}{4}$. They are most easily constructed by noting that when $\alpha = \beta = k$, we have $w = k$ also. We, therefore, draw a system of parallel lines through the points $\alpha = k$, $\beta = k$ with slope $-\frac{1}{4}$ and mark these with the corresponding value $w = k$ (Fig. 18*a*).

(2) The equation $pv^{1.41} = c$, for adiabatic expansion of certain gases where p = pressure and v = volume, can be written $\log p + 1.41 \log v = \log c$. If we choose $x = 10 \log v$, $y = 10 \log p$ for our perpendicular scales, the equations of the representing c-curves have the form $y + 1.41\, x = 10 \log c$, a set of parallel straight lines. These lines are easily constructed by noting that the slope is -1.41, and that through the point $v = 1$, $p = k$ there passes the line $c = k$ (Fig 18*a*).

(3) Consider the equation $f = \frac{4P}{\pi D^2}$, for the elastic limit of rivet steel, where P is the actual load in pounds at the elastic limit, D is the diameter

of the bar, and f is the fiber stress in pounds per square inch. If we choose $x = m_1P$, $y = m_2f$ for our perpendicular scales, the equations of the representing D-curves have the form $y = \frac{4\,m_2}{\pi m_1 D^2}\,x$, a set of radiating straight lines. For Fig. 18*b*, $m_1 = 2\,m_2$, and the lines were constructed by means

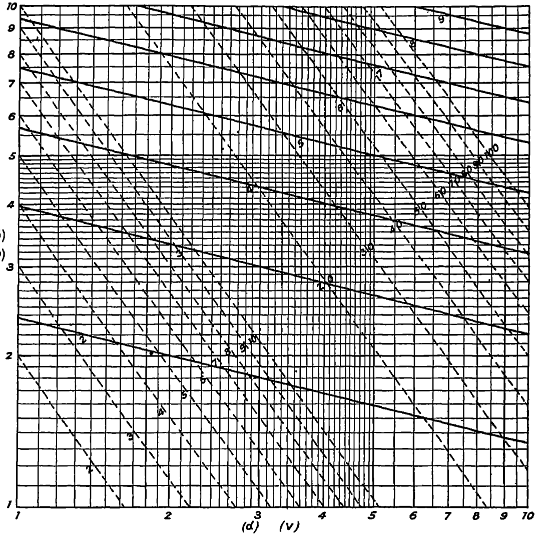
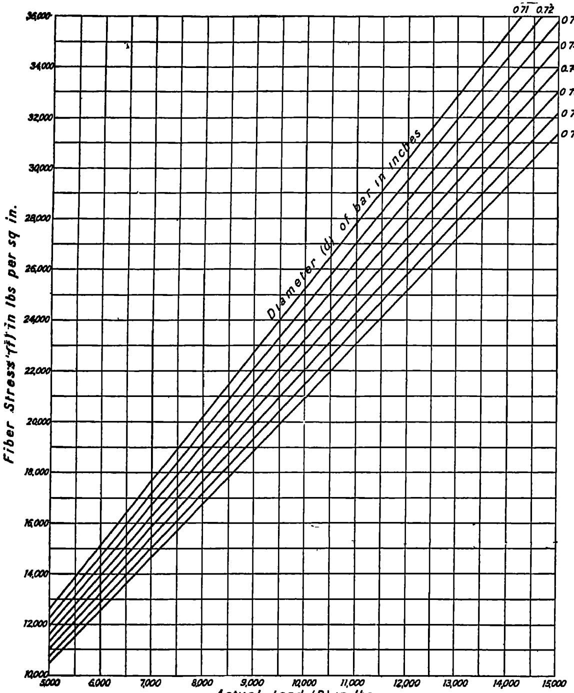


FIG. 18*a*.

of two points for values of $D = 0.71, 0.72, \ldots, 0.78$; thus for the line $D = 0.75$, we have $f = 2.26\,P$, and to construct this line we may use the points for which $P = 7000$, $f = 15{,}820$ and $P = 12{,}000$, $f = 27{,}120$.

We should note here that if we had chosen $x = m_1P$, $y = m_2D$ for our perpendicular scales, the equations of the representing f-curves would be $y^2 = \frac{4\,m_2^2}{\pi m_1 f}\,x$, a set of parabolas with a common vertex and a common axis; but these are much more difficult to draw than the straight lines above.



ELASTIC LIMIT OF RIVET STEEL $f = \frac{4P}{\pi d^2}$

FIG. 18*b*.

(4) Consider the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{w}$, where u and v are the distances of an object and its image from a lens and w is the focal length of the lens, or where w is the combined resistance of two resistances u and v in parallel. If we choose $x = m/u$, $y = m/v$ for our perpendicular scales,

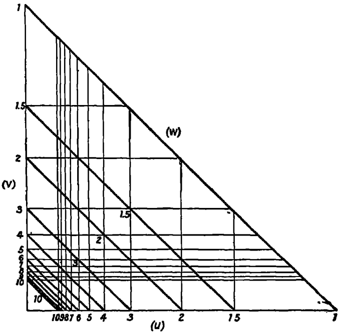


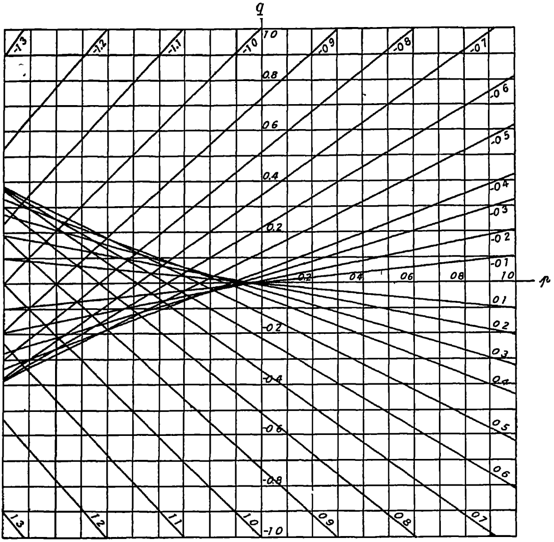
FIG. 18c.

the representing w-curves have for their equations $x + y = m/w$, a set of parallel straight lines. These lines are easily constructed by noting that the line marked $w = k$ joins the point $u = k$ on OX with the point $v = k$ on OY (Fig. 18c).

19. Rectangular chart for the solution of cubic equations. — The roots of the cubic equation $z^3 + pz + q = 0$ depend upon the values of the coefficients p and q; thus the roots are functions of p and q. If we choose $x = mp$, $y = mq$ for our perpendicular scales, the equations of our representing z-curves have the form $y + zx + mz^3 = 0$, a set of straight lines. In Fig. 19, these straight lines are constructed for values assigned to z, viz., $z = 0$, ± 0.1, ± 0.2, . . . , ± 1.3, and lying within a square bounded by $p = \pm 1$ and $q = \pm 1$. Each line is constructed by means of two points on it. Thus for the straight line marked $z = 0.3$, we have $0.027 + 0.3\,p + q = 0$, and to construct this line we may use the points for which $p = 1$, $q = -0.327$ and $p = -1$, $q = 0.273$.

On this chart we may read the approximate real roots of any cubic equation for which p and q lie within the limits -1 and $+1$. Thus for the equation $z^3 + 0.6\,z - 0.4 = 0$ we have $p = 0.6$ and $q = -0.4$, and we read $z = 0.47$, interpolating this value of z between the lines marked

$z = 0.4$ and $z = 0.5$. According as the point (p, q) falls outside of, on the boundary of, or within the triangular shaped region on the left, we can read one, two, or three values of z, and the corresponding cubic equation has one real root only, 3 real roots two of which are equal, or 3



RECTANGULAR CHART FOR SOLUTION OF CUBIC EQUATION

FIG. 19.

distinct real roots.* Thus for the equation $z^3 - 0.8z + 0.11 = 0$ we have $p = -0.8$ and $q = +0.11$, and we read $z = -0.96, +0.82, +0.14$.

If the values of p and q lie beyond the limits -1 and $+1$, the chart may still be used. Let $z = kz'$, and the equation $z^3 + pz + q = 0$ becomes $k^3z'^3 + pkz' + q = 0$, or $z'^3 + \frac{p}{k^2}z' + \frac{q}{k^3} = 0$, or $z'^3 + p'z' + q' = 0$. We may now choose k so that p' and q' lie within the limits -1 and $+1$, and read the corresponding values of z' from the chart. The roots of the

* The point (p, q) lies without, on the boundary of, or within the triangular shaped region according as $\frac{q^2}{4} + \frac{p^3}{27} \gtreqless 0$.

original equation are then $z = kz'$. Thus to solve the equation $z^3 + z - 4 = 0$, let $z = kz'$, and the equation becomes $z'^3 + \frac{1}{k^2} z' - \frac{4}{k^3} = 0$; if we choose $k = 2$, we get $z'^3 + 0.25\, z' - 0.5 = 0$, for which we read $z' = 0.69$, and hence $z = 1.38$.

If the complete cubic equation $u^3 + au^2 + bu + c = 0$ is given, this must first be transformed into the equation $z^3 + pz + q = 0$ by the substitution $u = z - \frac{a}{3}$.

In a similar way we may build a rectangular chart for the solution of the quadratic equation $z^2 + pz + q = 0$, or for any trinomial equation $z^m + pz^n + q = 0$.

20. Three-variable charts. Representing curves not straight lines.

(1) *Chart for chimney draft.* Extensive researches have been carried out by the Mechanical Engineering Department of the Massachusetts Institute of Technology to determine an equation expressing the draft of a chimney in terms of its height and the temperature of the flue gases. No simple relation between these quantities has been found. From the experiments performed, it was found that if T_1 is the absolute temperature in degrees Fahrenheit of the flue gases measured 3 feet above the center of the flue (the lowest temperature point recorded), H_2 is the height of the chimney in feet, and T_2 is the absolute temperature in degrees Fahrenheit of the flue gases at the top of the chimney, then

$$T_2 = \frac{T_1}{0.32\,(H_2 - 3)}\left[\left(\frac{H_2}{3}\right)^{0.96} - 1\right].$$

Now if D is the draft in inches of water, with the outside air at a temperature of 70° F., then

$$D = 0.192\left(0.075 - \frac{41.2}{T_2}\right)(H_2 - 3).$$

If the value of T_2 from the first of these equations is substituted in the second equation, we shall have an equation in three variables, D, T_1, and H_2.

In Fig. 20*a*, our two perpendicular scales are $x = m_1T_1$ and $y = m_2D$, where $m_1 = 200\,m_2$, and the representing H-curves are drawn for $H = 50, 75, \ldots, 300$ ft. Thus, for a chimney 150 ft. high and for an absolute temperature of 1139.5°, we read that the draft is 0.955 in. of water.

(2) *Experimental data* involving three variables are often plotted by means of a network of curves, and such a chart takes the place of a table of double entry. Fig. 20*b* gives a chart useful in heat flow problems where the temperature difference is an important factor. The chart gives the difference between the temperature of pure water under various gage pressures and the temperature under various vacuums. (Corrections must be applied for solutions.) Let P denote the gage pressure in

lbs. per sq. in., T, the temperature difference in degrees Fahrenheit, and V, the vacuum in inches. We first construct the perpendicular scales $x = m_1P$ and $y = m_2T$ (in Fig. 20b, $m_1 = 2\,m_2$); then the V-curves are constructed by means of a table, part of which is as follows:

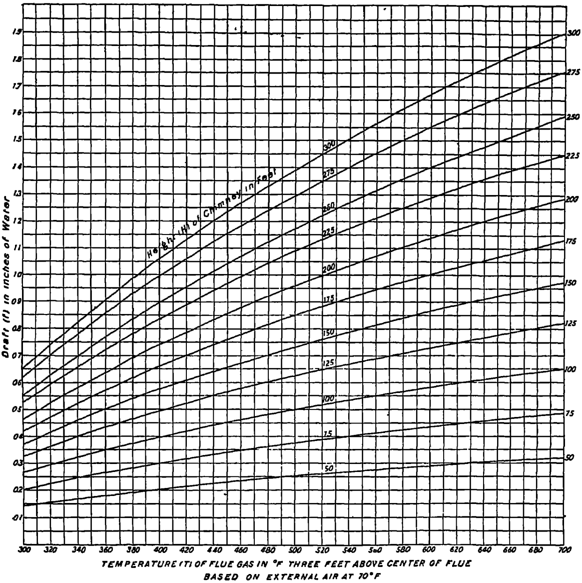


FIG. 20a.

TEMPERATURE DIFFERENCE (T).

Gage Pressure, P	Vacuum (V)			
	25	26	26½	....
0	78° 8	.	.	
5	93°.9	.	.	
10	106° 0	.	.	
15	.	.	.	
.	.		.	

Such a table is constructed with the aid of Peabody's Steam Tables. Thus, a vacuum of 25 in. is equivalent to a barometric pressure of 4.92 in. or $4.92 \times 0.4912 = 2.42$ lbs. per sq. in., and this gives a temperature of $133^\circ.2$; a gage pressure of 5 lbs. is equivalent to total pressure of 19.7 lbs. (adding the atmospheric pressure of 14.7 lbs.), and this gives a tempera-

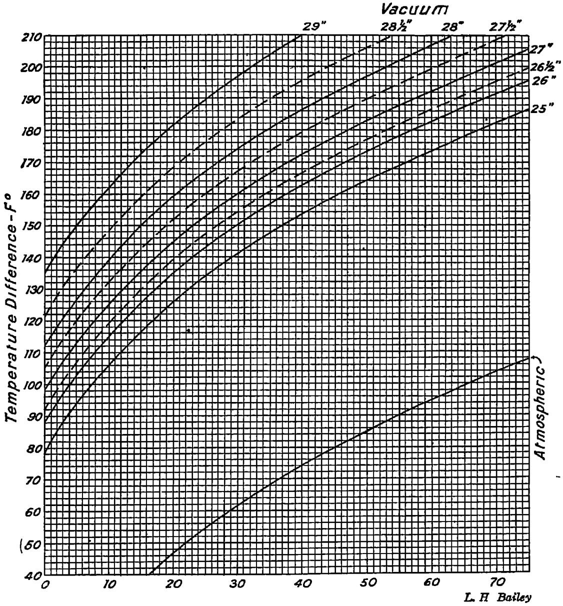


FIG 20*b*.

ture of $227^\circ.1$; we thus have a temperature difference of $227^\circ.1 - 133^\circ.2 = 93^\circ.9$. In this way, we subtract the temperature for a 25-in. vacuum from the temperatures at gage pressures of 0, 5, 10, . . . lbs., and these corresponding values of P and T are plotted giving the curve marked $V = 25$ in. The table is completed for various values of $V = 26$, $26\frac{1}{2}$, . . . in. and the corresponding curves are drawn. The curve

marked "atmospheric" gives the temperature difference for open evaporation.

21. Use of three indices. Hexagonal charts.—In Fig. 21*a* let OX and OY be perpendicular axes and let OZ be the bisector of their

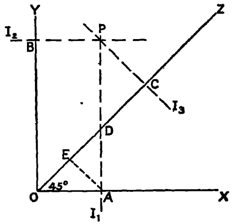


FIG. 21*a*.

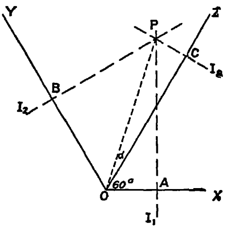


FIG. 21*b*.

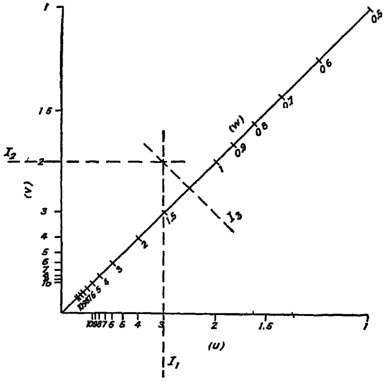


FIG. 21*c*.

angle. From any point P draw PI_1, PI_2, PI_3 perpendicular to OX, OY and OZ respectively, cutting OX in A, OY in B, OZ in C. Let PI_1 also cut OZ in D, and draw AE perpendicular to OD, cutting OZ in E. Then

$$OC = OE + ED + DC = OA\cos 45° + AD\cos 45° + DP\cos 45° = \frac{OA + OB}{\sqrt{2}}.$$

Thus, if the axes OX, OY, OZ carry the scales $x = mf(u)$, $y = mF(v)$, $z = \frac{m}{\sqrt{2}}\phi(w)$ respectively, then the three perpendiculars from any point to these axes will cut them so that

$$\frac{m}{\sqrt{2}}\phi(w) = \frac{mf(u) + mF(v)}{\sqrt{2}} \quad \text{or} \quad f(u) + F(v) = \phi(w);$$

thus, any equation of this type may be represented by three scales. The indices I_1, I_2, I_3 may be drawn on a transparent sheet and this sheet is

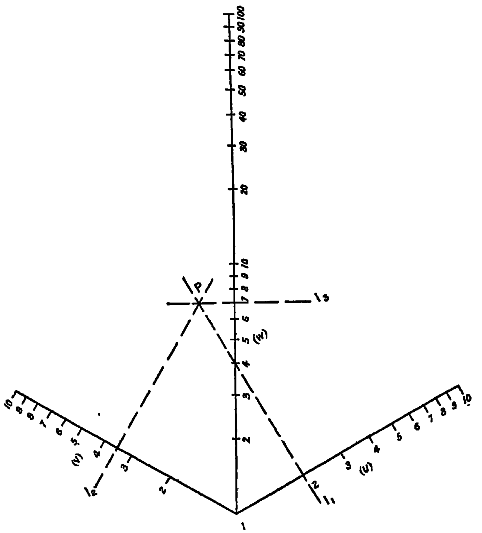


FIG. 21*d*.

moved over the paper keeping the indices perpendicular to the axes. Fig. 21*c* charts the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{w}$ by this method.

We can choose the modulus on OZ the same as the moduli for OX and OY by following the construction illustrated in Fig. 21*b*. Here OX and OY cut at an angle of 120° and OZ bisects this angle. Join OP and let angle $COP = \alpha$. Then

$$OA = OP \cos (60^\circ + \alpha) = OP (\cos 60^\circ \cos \alpha - \sin 60^\circ \sin \alpha).$$
$$OB = OP \cos (60^\circ - \alpha) = OP (\cos 60^\circ \cos \alpha + \sin 60^\circ \sin \alpha).$$
$$\therefore \quad OA + OB = 2\, OP \cos 60^\circ \cos \alpha = OP \cos \alpha = OC.$$

Thus our three scales are $x = mf(u)$, $y = mF(v)$, $z = m\phi(w)$. Fig. 21*d* charts the equation $uv = w$ or $\log u + \log v = \log w$ by this method.

EXERCISES.

1. Represent the equation $v = u^3$ by a straight line, using natural scales.

2. Represent the equations (a) $2u^2 + 3v^2 = 6$, (b) $u^2 - 3v^2 = 1$, (c) $u^2 + v^2 = 4$ by straight lines, using natural scales, and find graphically the simultaneous solutions of the three equations taken in pairs

3. Find graphically the simultaneous solutions of the equations $v = 6e^{-\frac{u^2}{16}}$ and $v = 10e^{-\frac{u^2}{9}}$.

4. Solve graphically the equation $u = 6e^{-\frac{u^2}{16}}$

5. Construct a sheet of logarithmic coördinate paper and draw on it the straight lines representing the relations

   (a) $v = u^3$; (b) $v = u^5$, (c) $v = \frac{1}{u^2}$;

   (d) $C = \pi D$ (circumference of circle), (e) $A = \frac{\pi}{4} D^2$ (area of circle);

   (f) $pv^{1.41} = 2$ (adiabatic expansion of a gas);

   (g) $h = \frac{v^2}{2g} = \frac{v^2}{64.4}$ (h = velocity head in ft., v = velocity in ft. per sec. for flow of water).

6. Construct a sheet of semilogarithmic coördinate paper and draw on it the straight lines representing the relations $v = 0.2\, e^{1.6u}$ and $v = 0.85\, (1.5)^{-3u}$.

7. Solve graphically the equation $u = 0.2\, e^{1.6u}$

8. Show how to solve for p the equation $\ln (pv^k) + \frac{b}{aR} pv = c$, where a, b, c, k, and R are known constants, when various values are assigned to v.

9. Construct charts for the relations $V = \pi r^2 h$ and $S = 2\pi rh$ (volume and lateral surface of a cylinder) using parallel straight lines only.

10. Plot the equation $y = 2x^n$ for various values of n, positive and negative, (a) as a set of curves, (b) as a set of straight lines.

11. Plot the equation $y = e^{nx}$ for various values of n, positive and negative, (a) as a set of curves, (b) as a set of straight lines.

12. Plot the following experimental data for the relative humidity obtained by a dew-point apparatus, using the wet bulb temperature, degrees Fahrenheit, as abscissas and the dry bulb temperature, degrees Fahrenheit, as ordinates.

WET BULB TEMPERATURE (DEG F)

Dry Bulb Temperature	Relative Humidity										
	0	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
40°	27.2	28 4	29 8	31 0	33 0	34 0	35 5	36 5	37 0	39 0	40 0
50°	32.7	34 7	36 5	38 5	40 7	42 7	44 0	45 5	47 0	48 7	50 0
60°	38.5	41 0	43 5	46 0	48 5	51 0	53 0	55 0	56 0	58 5	60 0
70°	44.0	47 2	50 3	53 0	56 0	59 0	61 5	63 5	66 0	68 0	70.0
80°	49 0	53 2	57 0	60.5	64 0	67 0	70 5	72 5	75 0	77 5	80 0
90°	54 0	58 8	64 0	68 0	72 0	76 0	79 0	81.5	84 0	87 5	90 0
100°	58 2	64 3	70 0	75 0	79 5	84 0	87 5	90 5	94 0	97 0	100 0
110°	62 0	69 3	76 0	82 0	87 5	92 0	96 0	100 0	103 0	106 7	110.0
120°	65 5	74 0	81 5	85 0	95 0	100 0	105 0	109 0	112 0	116 5	120 0
130°	69 0	78 6	87.0	95 2	103 0	109 0	112 0	119 0	122 0	126 0	130 0
140°	71 5	82 5	90 5	102 0	111 0	117 0	122 0	126 7	131 3	135 5	140.0

13. Construct a chart similar to that of Ex. 12, using the difference between wet and dry bulb temperatures as abscissas and the dry bulb temperature as ordinates. Which is the better representation, that of Ex. 12 or that of Ex. 13?

14. From the chart of Ex. 13, with the aid of graphical interpolation, form a table giving the relative humidity for dry bulb temperatures of 40°, 50°, . . . , 110°, and difference between wet and dry bulb temperatures of 0°, 5°, 10°, . . . , 45°, and draw the set of curves for the difference of temperatures using the dry bulb temperature as ordinates and the relative humidity as abscissas.

15. Plot the curves $y = a \sin bx$ for various values of a and b.

16. The Brake Horsepower of an engine ($B.H.P.$) with n cylinders of d in. diameter, according to the rating of the Association of Automobile Manufacturers, is given by $B.H.P. = \frac{d^2 n}{2.5}$. Plot the representative curves for $n = 2, 4, 6, 8, 12$, letting d range from $1\frac{1}{2}$ in. to 5 in., (a) using rectangular coördinate paper, (b) using logarithmic coordinate paper.

17. The volume, V, of one pound of superheated steam which has a pressure of P lbs. per sq. in. and a temperature of T degrees, is given by (Tumlirz's formula)

$$V = 0.596 \frac{T}{P} - 0.256 \text{ cu. ft.}$$

Plot representative lines (a) radiating, (b) parallel.

18. Solve the following equations by means of parabola and circle. (Art. 15.)

(a) $z^3 + 3z - 7 = 0$; (b) $z^3 + z + 5 = 0$;
(c) $z^3 - 3z^2 + 1 = 0$; (d) $z^4 - 12z + 7 = 0$,
(e) $z^4 + z - 1 = 0$; (f) $z^4 - 3z^3 + 3 = 0$.

19. Solve the following equation by means of the rectangular chart of Art. 19.

(a) $z^3 + 3z - 7 = 0$; (b) $z^3 + z + 5 = 0$;
(c) $z^3 - z^2 - 6z + 1 = 0$; (d) $z^3 + z^2 + z - 1 = 0$.

CHAPTER III.

NOMOGRAPHIC OR ALIGNMENT CHARTS.

22. Fundamental principle. — The methods employed in the preceding chapter for charting equations are very useful in a large number of problems in computation, but they have certain disadvantages: (1) the labor involved in their construction is great, especially when the representing curves are not straight lines; (2) the interpolation must largely be made between curves rather than along a scale, and thus accuracy is sacrificed; (3) the final charts appear very complex, especially if the methods are extended to equations involving more than three variables. The methods to be explained in this and the following chapters are applicable to a large number of equations or formulas and possess certain distinct advantages over the previous method: (1) the chart uses very few lines and is thus easily read; (2) interpolation is made along a scale rather than between curves, with a corresponding gain in accuracy; (3) the labor of construction is very small, thus saving time and energy; (4) the chart allows us to note instantly the change in one of the variables due to changes in the other variables

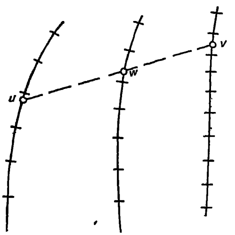


FIG. 22.

The fundamental principle involved in the construction of nomographic or alignment charts consists in the representation of an equation connecting three variables, $f(u, v, w) = 0$, by means of three scales along three curves (or straight lines) in such a manner that a straight line cuts the three scales in values of u, v, and w satisfying the equation. The transversal is called an *isopleth* or *index line* (Fig. 22).

We shall now make a study of some of the equations which can be represented in this way, and of the nature and relations of the scales representing the variables involved.*

* The principles underlying the construction of nomographic or alignment charts have been most fully developed by M D'Ocagne in his "Traité de Nomographie" Further references may be given to "The Construction of Graphical Charts," by J. B. Peddle; "Nomographic Solutions for Formulas of Various Types," by R. C. Strachan (Transactions of the American Society of Civil Engineers, Vol LXXVIII, p. 1359), and to various smaller articles that have appeared from time to time in Engineering Journals.

(I) EQUATION OF THE FORM $f_1(u) + f_2(v) = f_3(w)$ or $f_1(u) \cdot f_2(v) = f_3(w)$. — THREE PARALLEL SCALES.

23. **Chart for equation (I).** — [The second form of equation (I) can be brought immediately into the first form by taking logarithms of both members; thus, $\log f_1(u) + \log f_2(v) = \log f_3(w)$.]

Let AX, BY, CZ be three parallel axes with ABC any transversal or base line (Figs. 23*a*, 23*b*.) Draw any index line cutting the axes in

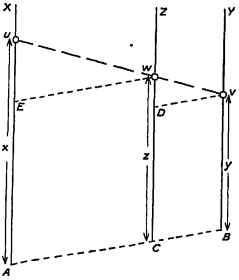


FIG. 23*a*.

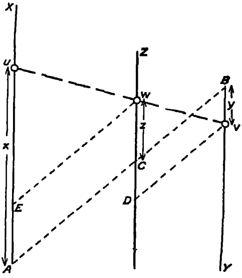


FIG. 23*b*.

the points u, v, w respectively, so that $Au = x$, $Bv = y$, $Cw = z$. How are x, y, z related?

If AC . $CB = m_1 : m_2$, and if through v and w we draw lines parallel to AB, then the triangles uEw and wDv are similar, and $Eu : Dw = Ew : Dv = AC : CB$ or $x - z : z - y = m_1 : m_2$.

$$\therefore \quad m_2x + m_1y = (m_1 + m_2)\,z \quad \text{or} \quad \frac{x}{m_1} + \frac{y}{m_2} = \frac{z}{\dfrac{m_1m_2}{m_1 + m_2}}.$$

Now if AX, BY, CZ carry the scales $x = m_1f_1(u)$, $y = m_2f_2(v)$, $z = \dfrac{m_1m_2}{m_1 + m_2}f_3(w)$, respectively, the last equation becomes $f_1(u) + f_2(v) = f_3(w)$, and any index line will cut the axes in three points whose corresponding values u, v, w satisfy this equation.

We also note that for the equation $f_1(u) - f_2(v) = f_3(w)$, the scales $x = m_1f_1(u)$ and $y = -m_2f_2(v)$ are constructed in opposite directions, as in Fig. 23*b*.

Hence to chart equation (I) $f_1(u) + f_2(v) = f_3(w)$, proceed as follows:

(1) Draw two parallel lines (x- and y-axes) any distance apart, and on these construct the scales $x = m_1f_1(u)$ and $y = m_2f_2(v)$, where m_1 and

m_2 are arbitrary moduli. The graduations of the u- and v-scales may start at any points on the axes.

(2) Draw a third line (z-axis) parallel to the x- and y-axes, such that (distance from x-axis to z-axis) : (distance from z-axis to y-axis) $= m_1 : m_2$.

(3) Determine a starting point for the graduations of the w-scale. This may be the point C ($z = 0$) cut out by the line from A ($x = 0$) to B ($y = 0$). If the range of the variables u and v is such that the points A and B do not appear on the scales, a starting point for the w-graduations may nevertheless be found by noting that three values of u, v, w satisfying equation (I) must be on a straight line; thus, assign values to u and v, say u_0 and v_0, and compute the corresponding value of w, say w_0, from equation (I); mark the point in which the line joining $u = u_0$ and $v = v_0$ cuts the z-axis with the value $w = w_0$ and use this last point as a starting point for the w-graduations.

(4) From the starting point for the w-graduations, construct the scale

$$z = m_3 f_3(w) = \frac{m_1 m_2}{m_1 + m_2} f_3(w).$$

General remarks. — In practice the index lines need not be drawn; a straight edge or a transparent sheet of celluloid with a straight line scratched on its under side or a thread can serve for reading the chart, *i.e.*, for finding the value of one of the variables when two of them are given. The distance between the outside scales and the moduli for these scales should, in general, be so chosen that the complete chart is almost square. Then any index line will cut the scales at an angle not less than 45°, and its points of intersection with the axes is more easily noted and the corresponding interpolation on the scales is more accurate. It is rarely necessary to choose the moduli so that the length of the longest scale greatly exceeds 10 inches.

Charts of logarithmic and uniform scales similar to those described in Art. 3 have been used in laying off the scales needed in the construction of most of the charts which follow. Much time and energy have been saved thereby. For greater convenience, the modulus of the primary or left-hand scale was taken to be 10 in. instead of 25 cm.

In laying off the w-scale with the help of these charts, the following procedure will increase the accuracy of the construction. Assign two or three sets of values to u and v, and compute the corresponding values of w; let these be (u_0, v_0, w_0), (u_1, v_1, w_1), and (u_2, v_2, w_2). Draw the index lines (u_0, v_0), (u_1, v_1), and (u_2, v_2), and mark the points in which these lines cut the z-axis with the corresponding values of w. Fold the chart along the scale with modulus m_3, and slide this scale along the z-axis until the points of the scale numbered w_0, w_1, w_2 practically coincide with the like-numbered points on the axis. This procedure is especially important when the modulus, m_3, is quite small.

The cuts in the text are reductions of the original drawings.

24. Chart for multiplication and division. The equation $u \cdot v = w$. — If we write this equation as $\log u + \log v = \log w$, we have an equation of the form (I). Let u and v range from 1 to 10; then w ranges from 1 to 100. Construct (Fig. 24*a*), 10 in. apart, the parallel scales $x = m_1 \log u = 10 \log u$ and $y = m_2 \log v = 10 \log v$. Since $m_1 : m_2 = 1 : 1$, the z-axis is midway between the x- and y-axes. The line joining $u = 1$ and $v = 1$ must cut the z-axis in $w = 1$, and using this last point as a starting point, construct the scale $z = \dfrac{m_1 m_2}{(m_1 + m_2)} w = 5 \log w$.

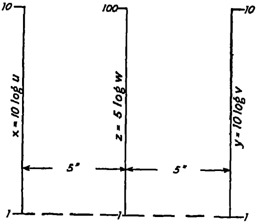


FIG. 24*a*.

The index line in the completed chart (Fig. 24*b*) gives the reading $u = 7$, $v = 3$, $w = 21$. Since the u- and v-scales are logarithmic scales, we may

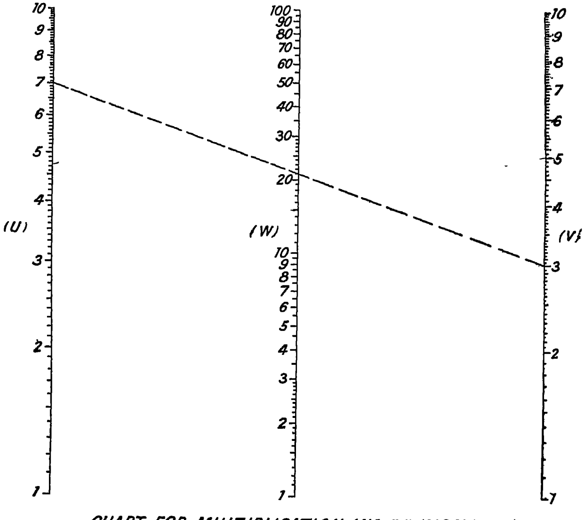


CHART FOR MULTIPLICATION AND DIVISION U V=W

FIG. 24*b*.

read these scales as ranging from 10^p to 10^{p+1} where p is any integer, with a corresponding change in the position of the decimal point in the value of w.

25. Combination chart for various formulas. — As a further illustration of product formulas of the type (I), $f_1(u) \cdot f_2(v) = f_3(w)$, we shall now represent several such formulas in the same chart, with the same outer scales but varying inner scale. Our chart (Fig. 25) represents six formulas and undoubtedly others could be added. In all cases, if m_1 and m_2 are the moduli of the scales on the x- and y-axes, then the modulus of the scale on the z-axis is $m_3 = m_1m_2/(m_1 + m_2)$, and the position of the z-axis is determined by the ratio $m_1 : m_2$.

(1) $\boldsymbol{u \cdot v = w}$ for multiplication and division. This has already been charted in Art. 24. The equations of the scales are

$$x = 10 \log u, \qquad y = 10 \log v, \qquad z = 5 \log w,$$

and $m_1 : m_2 = 1 : 1$. The index line gives the reading $u = 3$, $v = 5$, $w = 15$.

(2) $\boldsymbol{\sqrt[5]{u \cdot v^4} = w}$ occurs in the McMath "run-off" formula. The equation can be written $\log u + 4 \log v = 5 \log w$ and hence

$$x = m_1 \log u, \qquad y = m_2 (4 \log v), \qquad z = m_3 (5 \log w).$$

Let $m_1 = 10$ and $m_2 = 10/4$, then $m_3 = 2$, and $m_1 : m_2 = 4 : 1$. The equations of our scales are

$$x = 10 \log u, \qquad y = 10 \log v, \qquad z = 10 \log w.$$

A starting point for the w-scale is found by noting that when $u = 1$ and $v = 1$ then $w = 1$, and by aligning these three points. The index line gives the reading $u = 3$, $v = 5$, $w = 4.5$.

(3) $\boldsymbol{pv^{1.41} = c}$ gives the pressure-volume relation of certain gases under adiabatic expansion. The equation can be written $\log p + 1.41 \log v = \log c$, hence

$$x = m_1 \log p, \qquad y = m_2 (1.41 \log v), \qquad z = m_3 \log c.$$

If we choose $m_1 = 10$ and $m_2 = 10/1.41$, then $m_3 = 4.15$ and $m_1 : m_2 = 1.41 : 1$. The equations of our scales are

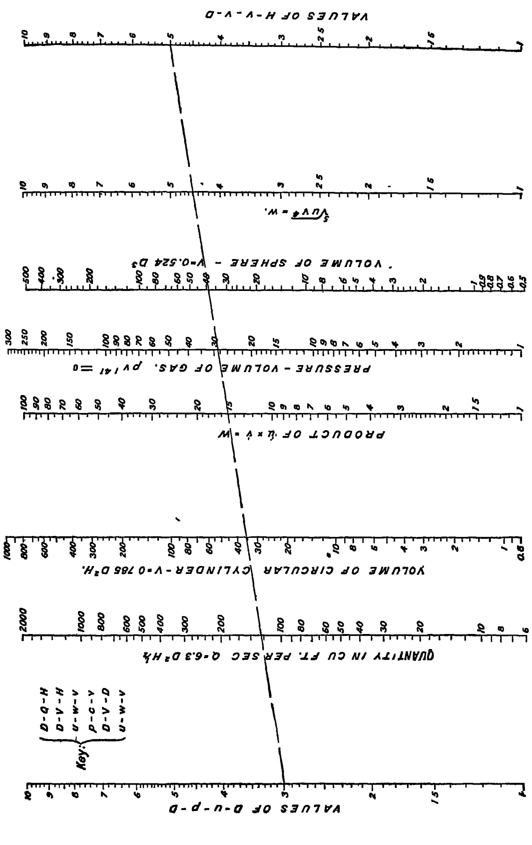
$$x = 10 \log p, \qquad y = 10 \log v, \qquad z = 4.15 \log c.$$

A starting point for the c-scale is found by noting that when $p = 1$ and $v = 1$ then $c = 1$ and by aligning these three points. The index line gives the reading $u = 3$, $v = 5$, $w = 29$.

(4) $\boldsymbol{V = 0.785\, D^2H}$, the volume of a circular cylinder. The equation can be written $2 \log D + \log H = (\log V - \log 0.785)$, hence

$$x = m_1 (2 \log D), \qquad y = m_2 \log H, \qquad z = m_3 (\log V - \log 0.785).$$

If we choose $m_1 = 5$ and $m_2 = 10$, then $m_3 = 3.33$ and $m_1 : m_2 = 1 : 2$.



COMBINATION CHART.

FIG. 25.

The equations of our scales are

$$x = 10 \log D, \qquad y = 10 \log H, \qquad z = 3.33 \log V,$$

where we have discarded the expression $-3.33 \log 0.785$ in the value of z, this may be done since this expression merely helps to determine a starting point for the V-scale; thus the point $V = 1$ is at a vertical distance $z = -3.33 \log 0.785$ from the base line AB of Fig. 23a or 23b. We shall however determine a starting point for the V-scale by noting that when $D = 1$ and $H = 1$, then $V = 0.785$, and by aligning these three points. The index line gives the reading $D = 3$, $H = 5$, $V = 35$.

(5) $V = 0.524\, D^3$, **the volume of a sphere.** The equation can be written $V = 0.524\, D \cdot D^2$ or $\log D + 2 \log D = \log V - \log 0.524$, hence

$$x = m_1 \log D, \qquad y = m_2 (2 \log D), \qquad z = m_3 (\log V - \log 0.524).$$

If we choose $m_1 = 10$ and $m_2 = 5$, then $m_3 = 3.33$ and $m_1 : m_2 = 2 : 1$. The equations of our scales are

$$x = 10 \log D, \qquad y = 10 \log D, \qquad z = 3.33 \log V,$$

where we have discarded the expression $-3\ 33 \log 0.524$ in the value of z. We find a starting point for the V scale by noting that when $D = 1$, $V = 0.524$ and we align the three points $D = 1$, $D = 1$, and $V = 0.524$.

(6) $Q = 6.3\, D^2 \sqrt{H}$ gives the quantity of water, Q, in cu. ft. per second which flows through a pipe having a diameter D ft. when under a head H feet. The equation can be written $2 \log D + \frac{1}{2} \log H = (\log Q - \log 6.3)$. If we choose $m_1 = 5$ and $m_2 = 20$, then $m_3 = 4$ and $m_1 : m_2 = 1 : 4$. The equations of our scales are

$$x = 10 \log D,$$
$$y = 10 \log H,$$
$$z = 4 \log Q.$$

Again we discard the expression $-4 \log 6.3$ in the value of z, and find a starting point for the Q-scale by noting that when $D = 1$ and $H = 1$, then $Q = 6.3$ and by aligning these three points. The index line gives the reading $D = 3$, $H = 5$, $Q = 127$.

26. Grashoff's formula $w = 0.0165\, AP_1^{0.97} = 0.01296\, D^2 P_1^{0.97}$ **for the weight,** w, **of dry saturated steam in** pounds per second flowing from a reservoir at pressure P_1 pounds per sq. in. through a standard converging orifice of A sq. in. or circular orifice of diameter D in. to a pressure of P_2 pounds per sq. in., if $P_1 \geqq 0.6\, P_2$.

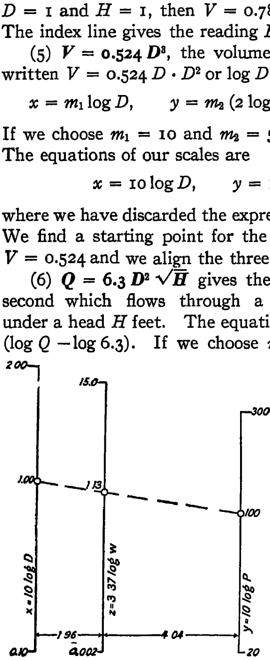
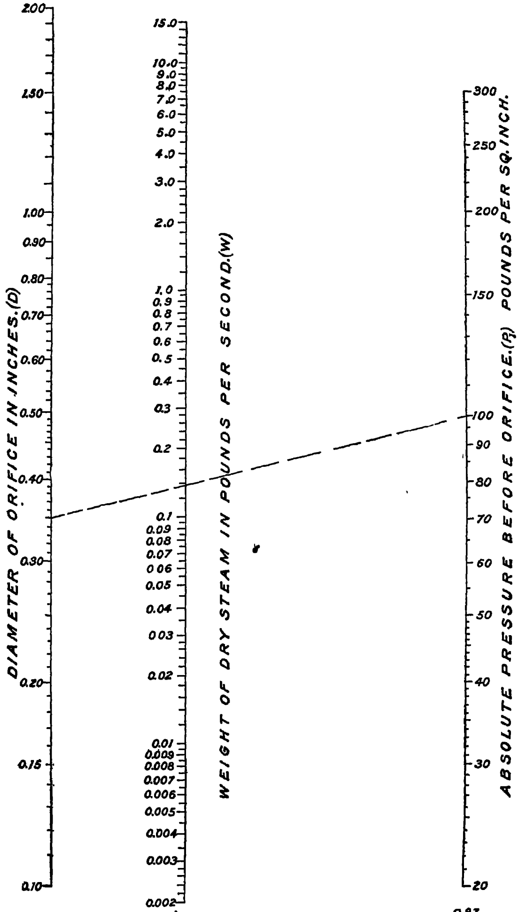


FIG. 26a.



GRASHOFF'S FORMULA $W = 0.0165\,A\,P_1^{0.97}$

Fig. 26*b*.

If we write the equation as

$$2 \log D + 0.97 \log P_1 = (\log w - \log 0.01296)$$

we have an equation of the form (I). The scales are

$$x = m_1 (2 \log D), \quad y = m_2 (0.97 \log P_1), \quad z = m_3 (\log w - \log 0.01296).$$

Let D vary from 0.1 in. to 2.0 in., then $\log D$ varies from $\log 0.1 = -1$ to $\log 2 = 0.301$, a range of 1.301; if we choose $m_1 = 5$, the equation of the D-scale will be $x = 10 \log D$ and the scale will be about 13 in. long. Let P_1 vary from 20 pounds to 300 pounds, then $\log P_1$ varies from $\log 20 = 1.301$ to $\log 300 = 2.477$, a range of 1.176, if we choose $m_2 = 10/0.97$, the equation of the P-scale will be $y = 10 \log P$ and the scale will be about 12 in. long. Then $m_3 = m_1 m_2/(m_1 + m_2) = 3.37$. The equations of our scales are now

$$x = 10 \log D, \qquad y = 10 \log P, \qquad z = 3.37 \log w.$$

Construct (Fig. 26*a*) the x- and y-axes 6 in. apart; the z-axis will divide this distance in the ratio $m_1 \cdot m_2 = 4.85 \cdot 10$. We have therefore drawn the z-axis at a distance of 1.96 in. from the x-axis and 4.04 in. from the y-axis. A starting point for the w-scale is found by aligning $D = 1$, $P_1 = 100$, and $w = 1.13$. The completed chart is given in Fig. 26*b*.

27. Tension in belts, $\dfrac{T_2}{T_1} = e^{-0.01745 f\alpha}$, and horsepower of belting, $H.P. = \dfrac{(T_1 - T_2)\,S}{33{,}000}$.—In the first of these formulas, T_1 is the allowable working stress in pounds per in. of width, or the tension in the tight side of the belt; the value of this may be obtained either from the manufacturer of the belt or by breaking a piece in a tension machine; a suitable factor of safety should be added. T_1 may vary from about 50 to 75 for single belts and from 100 to 150 for double belts. α is the arc of contact in degrees of belt and pulley and may vary from 100° to 300°. f is the coefficient of friction and is assumed (in this chart) to have the value 0.30 for leather belts on cast-iron pulleys. T_2 is the tension in the loose side of the belt in pounds per in. of width. This formula may be written

$$\log T_2 - \log T_1 = -0.01745 f\alpha \log e \quad \text{or} \quad \log T_1 - 0.002274\,\alpha = \log T_2$$

which is in the form (I). The scales are

$$x = m_1 \log T_1, \qquad y = -m_2 (0.002274\,\alpha), \qquad z = m_3 \log T_2.$$

Now $\log T_1$ varies from $\log 50 = 1.6990$ to $\log 150 = 2.1761$, a range of 0.4771; if we choose $m_1 = 10$, the equation of the T_1-scale will be $x = 10 \log T_1$ and the scale will be about 5 in. long. Again, α has a range of 200; if we choose $m_2 = \dfrac{1}{40\,(0.002274)}$, the equation of the α-scale will be

$y = -\frac{1}{40}\alpha$ and the scale will be 5 in. long. Then $m_3 = m_1m_2/(m_1 + m_2) = 5.24$ The equations of our scales are

$$x = 10 \log T_1, \qquad y = -\tfrac{1}{40}\alpha, \qquad z = 5.24 \log T_2.$$

Construct (Fig. 27*a*) the x- and y-axes 3.75 in. apart; the z-axis must divide this distance in the ratio $m_1 : m_2 = 10 : \dfrac{1}{40\,(0.002274)} = 10 : 11$. We have therefore drawn the z-axis at a distance of 1.79 in. from the x-axis and 1.96 in. from the y-axis. A starting point for the T_2 scale is found by aligning $T_1 = 80$, $\alpha = 150$, and $T_2 = 36.5$. The completed chart is given in Fig. 27*b*, and indicates the reading $T_1 = 80$ pounds, $\alpha = 150°$, $T_2 = 36.5$ pounds.

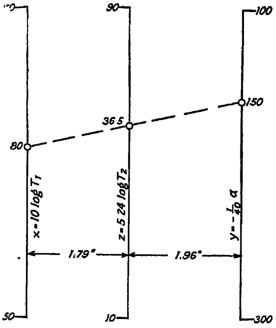


FIG. 27*a*.

In the second of the formulas, S is the distance traveled by the belt in feet per minute, and may vary from 300 to 6000; $T_1 - T_2$ is the difference in the tensions and may vary from 10 to 200; *H.P.* is the horsepower which a belt of one inch width will transmit; then, knowing the horsepower which we wish to transmit we merely divide to get the width of the belt desired. The equation can be written

$$\log (T_1 - T_2) + \log S = \log H.P. + \log 33{,}000,$$

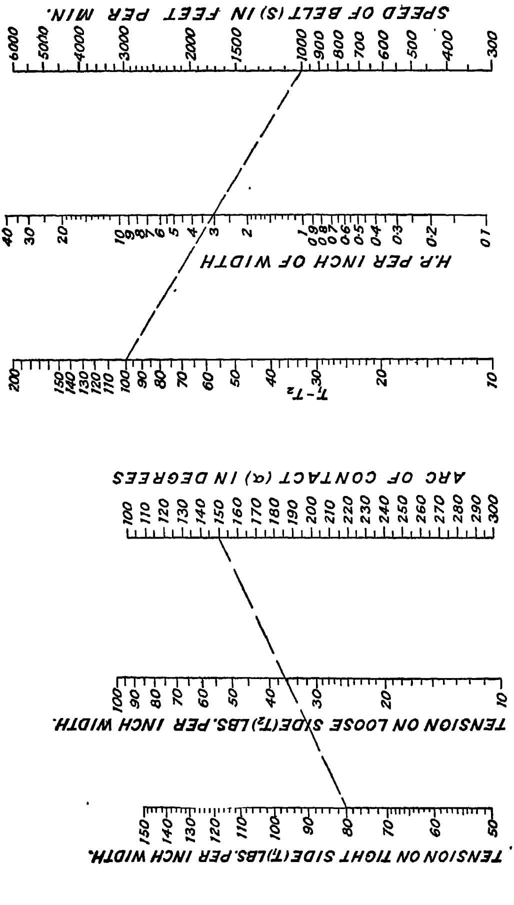
which has the form (I). The scales are

$$x = m_1 \log (T_1 - T_2), \qquad y = m_2 \log S, \qquad z = m_3 \log H.P.$$

Now $\log (T_1 - T_2)$ varies from $\log 10 = 1$ to $\log 200 = 2.3010$, a range of 1.3010; if we choose $m_1 = 5$, the equation of the $(T_1 - T_2)$-scale will be $x = 5 \log (T_1 - T_2)$ and the scale will be about 6.5 in. long. $\log S$ varies from $\log 300 = 2.4771$ to $\log 6000 = 3.7781$, a range of 1.3010; if we choose $m_2 = 5$, the equation of the S-scale will be $y = 5 \log S$ and the scale will be about 6 5 in. long. Then $m_3 = m_1m_2/(m_1 + m_2) = 2.5$. The equations of our scales are

$$x = 5 \log (T_1 - T_2), \qquad y = 5 \log S, \qquad z = 2.5 \log H.P.$$

Construct the x- and y-axes 4 in. apart; the z-axis must divide this distance in the ratio $m_1 : m_2 = 1 : 1$. A starting point for the *H.P.* scale is found by aligning $T_1 - T_2 = 10$, $S = 300$, and $H.P. = 0.091$. The completed chart is given in Fig. 27*c* and indicates the reading $T_1 - T_2 = 100$, $S = 1000$, $H P. = 3.0$.



H.P. OF BELTS.

$$H.P. = \frac{(T_1 - T_2)S}{33000}$$

TENSION IN BELTS.

$$\frac{T_2}{T_1} = e^{-0.01745 f \alpha}$$

FIGS. 27b, 27c.

(II) EQUATION OF FORM $f_1(u) + f_2(v) + f_3(w) + \cdots = f_4(t)$ or $f_1(u) \cdot f_2(v) \cdot f_3(w) \cdots = f_4(t)$. FOUR OR MORE PARALLEL SCALES.

28. Chart for equation (II). — [The second form of equation (II) can be brought immediately into the first form by taking logarithms of both members; thus $\log f_1(u) + \log f_2(v) + \log f_3(w) + \cdots = \log f_4(t)$.] Equation (II) is merely an extension of equation (I) and the method of charting the former is an extension of the method employed in charting the latter.

For definiteness, let us consider the case of four variables and the equation in the form $f_1(u) + f_2(v) + f_3(w) = f_4(t)$. Let $f_1(u) + f_2(v) = q$. This equation is in the form (I) and can therefore be charted by means

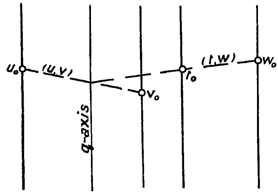


FIG. 28a.

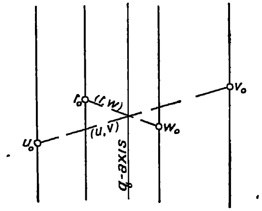


FIG. 28b.

of three parallel scales, but the q-scale need not be graduated. (Fig. 28*a*.) We then have $q + f_3(w) = f_4(t)$, which is also in the form (I) and can therefore be charted by means of three parallel scales one of which is the q-scale already constructed. The graduations of the u-, v-, and w-scales may start anywhere along their axes, but a starting point for the graduations of the t-scale must be determined by a set of values $u = u_0$, $v = v_0$, $w = w_0$, $t = t_0$ satisfying equation (II); thus, join u_0 and v_0 by a straight line and mark its point of intersection with the q-axis; join this point with w_0 cuttfng the t-scale in a point which must be marked t_0; this last point is then used as a starting point for constructing the t-scale. To read the completed chart we thus use *two index lines*, one joining points on the u- and v-scales, the other joining points on the w- and t-scales, intersecting the q-axis in the same point. Fig. 28*a* illustrates the position of the scales. It is thus easy to find the value of any one of the four variables when the other three are known.

The extension of this method to equations of the form (II) containing more than four variables is obvious.

Considering again the case of four variables, $f_1(u) + f_2(v) + f_3(w) = f_4(t)$, we can write this in the form $f_1(u) + f_2(v) = f_4(t) - f_3(w) = q$ and chart each of the equations $f_1(u) + f_2(v) = q$ and $f_4(t) - f_3(w) = q$ by means of three parallel scales, with the q-scale (which is not graduated) in common. Again, to read the chart, we use two index lines, one joining points on the u- and v-scales and the other joining points on the w- and t-scales, intersecting the q-axis in the same point Fig. 28b illustrates the position of the scales in this case.

29. Chezy formula for the velocity of flow of water in open channels, $v = c\sqrt{rs}$. — Here, v is the velocity of flow in ft. per sec., r is the hydraulic radius in ft. (area divided by wetted perimeter), s is the slope of the water surface, and c is a coefficient depending on the condition of the channel. (See Art. 53 for the construction of a chart computing c by the Bazin formula.)

Let our variables range as follows: s from 0.00005 to 0.01, r from 0.1 ft. to 20 ft., c from 10 to 250. Writing the equation

$$\tfrac{1}{2}\log s + \tfrac{1}{2}\log r + \log c = \log v$$

we have an equation of the form (II). Introducing an auxiliary quantity, q, we can write

$$(1)\ \tfrac{1}{2}\log s + \tfrac{1}{2}\log r = q \quad \text{and} \quad (2)\ q + \log c = \log v.$$

We now construct a chart for the first of these equations. The scales are

$$x = m_1(\tfrac{1}{2}\log s), \qquad y = m_2(\tfrac{1}{2}\log r), \qquad z = m_3 q.$$

Now $\log s$ varies from $\log 0.00005 = 5.6990 - 10$ to $\log 0.01 = 8.0 - 10$, a range of 2.3010; and if we choose $m_1 = 10$, the equation of the s-scale is $x = 5\log s$ and the scale will be about 11.5 in. long. Again, $\log r$ varies from $\log 0.1 = -1$ to $\log 20 = 1.3010$, a range of 2.3010; and if we choose $m_2 = 10$, the equation of the r-scale is $y = 5\log r$ and the scale will be about 11.5 in. long. Then $m_3 = m_1m_2/(m_1 + m_2) = 5$. The equations of our scales are

$$x = 5\log s, \qquad y = 5\log r, \qquad z = 5\,q.$$

Construct (Fig. 29a) the x- and y-axes at any convenient distance, say 8 in. apart; the z-axis must divide this distance in the ratio $m_1 : m_2 = 1 : 1$, and hence the z-axis is drawn midway between the x- and y-axes. The q-scale need not be graduated.

We continue the construction by charting the second equation. The scales are

$$z = m_3 q, \qquad a = m_4 \log c, \qquad b = m_5 \log v.$$

We use the same q-scale as above so that $m_3 = 5$. Log c varies from $\log 10 = 1$ to $\log 250 = 2.3979$, a range of 1.3979; and if choose $m_4 = 5$, the length of the scale will be about 7 in. Then $m_5 = m_3m_4/(m_3 + m_4) = 2.5$. The equations of our scales are

$$z = 5\,q, \qquad a = 5\log c, \qquad b = 2.5\log v.$$

Construct (Fig. 29*a*) the *a*-axis at any convenient distance, say 10 in. from the *z*-axis. The graduations of the *c*-scale may start anywhere along the *a*-axis; for symmetry, we shall place the scale opposite the middle of the scales already constructed. The *b*-axis must divide the distance between the *z*- and *a*-axes in the ratio $m_3 : m_4 = 1 : 1$, and it is therefore drawn midway between them. We get a starting point for the *v*-scale by making a single computation; thus, when $s = 0.001$, $r = 1$ and $c = 100$, we have $v = 3.16$; hence, join $s = 0.001$ and $r = 1$, cutting the *q*-axis in a point, and then join this point and $c = 100$, cutting the *b*-axis in a point which must be marked $v = 3.16$. Starting at this last point and proceeding along the axis, the *v*-scale is graduated from $v = 0.02$ to $v = 100$.

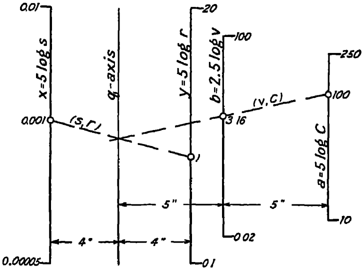


FIG. 29*a*.

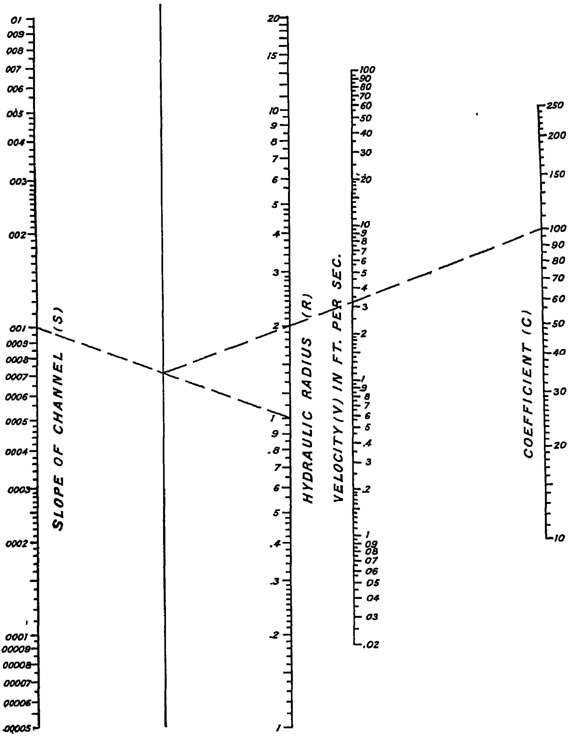
The completed chart is given in Fig. 29*b*. To read the chart we need merely remember that the (s, r) and (c, v) index lines must intersect on the *q*-axis. The index lines drawn in Fig. 29*b* show that when $s = 0.001$, $r = 1$ ft. and $c = 100$, then $v = 3.16$ ft. per sec.

30. **Hazen-Williams formula for the velocity of flow of water in pipes, $V = CR^{0.63}S^{0.54}(0.001)^{-0.04}$. — The quantity of water discharged, $Q = 4\pi R^2 V$.** — The first of these formulas has been derived experimentally by Hazen and Williams, who have also constructed a slide rule for its solution. V is the velocity of discharge in ft. per sec. from circular pipes or channels flowing full; R is the hydraulic radius in ft. (area of cross-section divided by wetted perimeter); S is the slope or ratio of rise to length of pipe; C is a coefficient depending on the material and the condition of the inner surface of the pipe. Williams and Hazen give the following table of values for C:

Brass, block tin, lead, glass ..	140–150	Cast iron, old, bad condition.....	60–100
Cast iron, very smooth	.140–145	Steel pipe, riveted, new	105–115
" " new, good condition	.125–135	" " " old	90–105
" " old, " "	. 100–125	Masonry conduits.... . ..	110–135

Replacing $(0.01)^{-0.04}$ by its value 1.318 and expressing R in inches instead of in feet, the formula becomes $V = 0.2755\ CR^{0.63}S^{0.54}$, and this can be written as

$$0.63 \log R + 0.54 \log S + \log C + \log 0.2755 = \log V$$



$$V = C\sqrt{RS}$$

FIG. 29*b*.

which is of the form (II). Let our variables range as follows: R from 0.1 in. to 20 in., S from 0.0001 to 0.05, C from 25 to 200.

We first construct a chart for $0.63 \log R + 0.54 \log S = q$. The scales are

$$x = m_1(0.63 \log R), \qquad y = m_2(0.54 \log S), \qquad z = m_3 q.$$

Now $\log R$ varies from $\log 0.1 = -1$ to $\log 20 = 1.3010$, a range of 2.3010; if we choose $m_1 = 5/0.63$, the equation of the R-scale will be $x = 5 \log R$ and the scale will be about 11.5 in. long. Again, $\log S$ varies from $\log 0.0001 = 6 - 10$ to $\log 0.05 = 8.6990 - 10$, a range of 2.6990; if we choose $m_2 = 5/0.54$, the equation of the S-scale will be $y = 5 \log S$ and the scale will be about 13.5 in. long. Then $m_3 = m_1m_2/(m_1 + m_2) = 4.27$. The equations of our scales are

$$x = 5 \log R,$$
$$y = 5 \log S,$$
$$z = 4.27\, q.$$

Construct (Fig. 30a) the x- and y-axes 11.7 in. apart; the z-axis must divide this distance in the ratio $m_1 : m_2 = \dfrac{5}{0.63} : \dfrac{5}{0.54} = 5.4 : 6.3$. We have therefore drawn the z-axis at a distance of 5.4 in. from the x-axis and 6.3 in. from the y-axis. The q-scale need not be graduated.

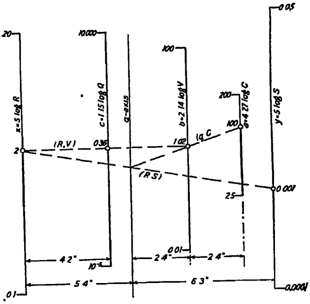


FIG. 30a.

We now continue the construction by charting

$$q + \log C + \log 0.2755 = \log V.$$

The scales are

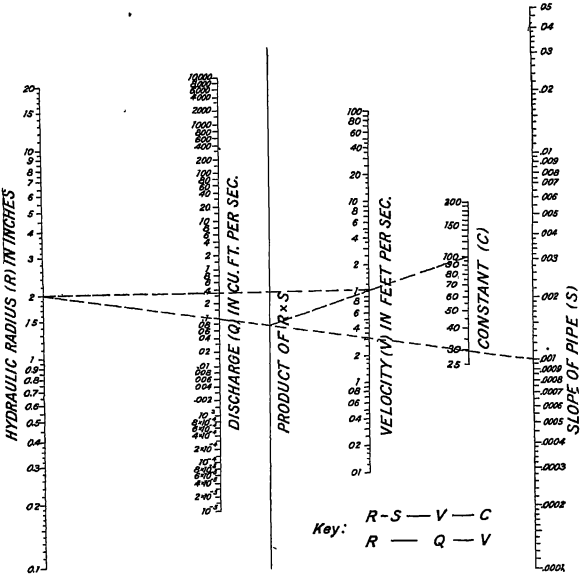
$$z = m_3 q, \qquad a = m_4(\log C + \log 0.2755), \qquad b = m_5 \log V.$$

We use the same q-scale as above so that $m_3 = 4.27$. Log C varies from $\log 25 = 1.3979$ to $\log 200 = 2.3010$, a range of 0.9031; if we choose $m_4 = 4.27$, the equation of the C-scale will be $a = 4.27 \log C$ and the scale will be about 4 in. long. Then $m_5 = m_3m_4/(m_3 + m_4) = 2.14$. The equations of our scales are

$$z = 4.27\, q, \qquad a = 4.27 \log C, \qquad b = 2.14 \log V.$$

Construct (Fig. 30a) the a-axis at a distance of 4.8 in. from the z-axis. Although the graduations of the C-scale may start anywhere along the a-axis, the C-scale is only about 4 in. long and we shall get a more symmetrical chart by placing the scale opposite the middle of the S-scale.

The b-axis must divide the distance between the z-axis and a-axis in the ratio $m_3 : m_4 = 1 : 1$, and is therefore midway between them. We get a starting point for the V-scale by making a single computation, thus, when $R = 2$, $S = 0.001$ and $C = 100$, we have $V = 1.02$; hence, join $R = 2$ and $S = 0.001$, cutting the q-axis in a point, and then join this point and $C = 100$, cutting the b-axis in a point which must be marked $V = 1.02$.



HAZEN AND WILLIAMS FORMULA FLOW OF WATER IN PIPES
$V = CR^{0.63}S^{0.54}0.001^{0.04}$ $Q = AV = 4\pi R^2 V.$

FIG 30b.

We shall now enlarge the usefulness of our chart by adding a scale for Q, the quantity discharged in cu. ft. per sec. For circular pipes, we have $Q = 4\pi R^2 V$, where V is the velocity of discharge in ft. and R is the hydraulic radius (one-fourth of the diameter of the pipe) in ft., or $Q = 0.0873\, R^2 V$, where R is expressed in inches. We write this equation in the form

$$2 \log R + \log V = (\log Q - \log 0.6873),$$

and our scales are

$$x = m_1 (2 \log R), \qquad b = m_2 \log V, \qquad c = m_3 (\log Q - \log 0.0873).$$

As we want to use the R-scale already constructed, we take $m_1 = 5/2$, and the equation of the scale is $x = 5 \log R$, as above. We also want to use the V-scale already constructed and hence take $m_2 = 2.14$ and get $b = 2.14 \log V$, as above. Then $m_3 = m_1m_2/(m_1 + m_2) = 1.15$, and the equation of the Q-scale is $c = 1.15 \log Q$. In Fig. 30*a*, the x- and b-axes are 7.8 in. apart. The c-axis must divide this distance in the ratio $m_1 : m_2 = 2.5 : 2.14$; this is accomplished by drawing the c-axis at a distance of 4.2 in. from the x-axis. We get a starting point for the Q-scale by aligning $R = 2$, $V = 1.02$, and $Q = 0.36$.

Fig. 30*b* gives the completed chart. The index lines drawn indicate that when $R = 2$ in., $S = 0.001$, and $C = 100$, then $V = 1.02$ ft. per sec. and $Q = 0.36$ cu. ft. per sec.

31. Indicated horsepower of a steam engine, $H.P. = \dfrac{PLAN}{33{,}000}$.— Here, P is the mean effective pressure in pounds per sq. in., L is the length of the stroke in ft., A is the area of the piston in sq. in, and N is the speed in revolutions per minute.

This formula is used extensively in steam engine testing practice. The pressure, P, is obtained from the indicator card and is equal to its area divided by its length. In double-acting steam engines — air compressors, air engines, or water pumps — we have the fluid acting on both sides of the piston alternately. Here we must apply the formula to each end and add the results in order to get the total power output.

For purposes of illustration we shall here use the diameter, D, instead of the area, and write

$$H.P. = \frac{\pi PLD^2N}{(33{,}000)\,(4)\,(12)} = 0.000001983\, PLD^2N$$

where L is expressed in inches, as is more common.

We shall divide the charting of this equation into three parts:

$$(1)\ PL = q, \qquad (2)\ D^2N = t, \qquad (3)\ H.P. = 0.000001983\, qt.$$

(1) $PL = q$ can be written $\log P + \log L = \log q$, which has the form (I), and our scales are

$$x = m_1 \log P, \qquad y = m_2 \log L, \qquad z = m_3 \log q.$$

If P varies from 10 to 200, $\log P$ varies from 1 to 2.3010, a range of 1.3010; if we choose $m_1 = 10$, the P-scale will be about 13 in. long. If L varies from 2 to 40, $\log L$ varies from 0.3010 to 1.6020, a range of 1.3010; if we choose $m_2 = 10$, the L scale will be about 13 in. long. Then $m_3 = m_1m_2/(m_1 + m_2) = 5$, and the equations of our scales are

$$x = 10 \log P, \qquad y = 10 \log L, \qquad z = 5 \log q.$$

Construct (Fig. 31*a*) the x- and y-axes 10 in. apart. The z-axis must divide this distance in the ratio $m_1 : m_2 = 1 : 1$ and is therefore drawn midway between them, but the q-scale need not be graduated.

(2) $D^2N = t$ can be written $2 \log D + \log N = \log t$, which has the form (I), and our scales are

$$a = m_4 (2 \log D), \qquad b = m_5 \log N, \qquad c = m_6 \log t.$$

If D varies from 2 to 40, $\log D$ varies from 0.3010 to 1.6020, a range of 1.3010; and if we choose $m_4 = 5$, the equation of the D-scale will be $a = 10 \log D$ and the scale will be about 13 in. long. If N varies from 50 to 1000, $\log N$ varies from 1.6990 to 3.0000, a range of 1.3010; and if we choose $m_5 = 10$, the N-scale will be about 13 in. long. Then $m_6 = m_4m_5/(m_4 + m_5) = 3.33$, and the equations of our scales are

$$a = 10 \log D, \qquad b = 10 \log N, \qquad c = 3.33 \log t.$$

Construct (Fig. 31*a*) the a- and b-axes 10 in. apart. Since D and L have the same range and their scales have the same modulus, for $y = 10 \log L$ and $a = 10 \log D$, we find it convenient to make the a-axis coincide with the y-axis, and to use one scale for both L and D. The c-axis must divide the distance between the a- and b-axes in the ratio $m_4 : m_5 = 1 : 2$, or the c-axis is at a distance of 3.33 in. from the a-axis, but the t-scale need not be graduated.

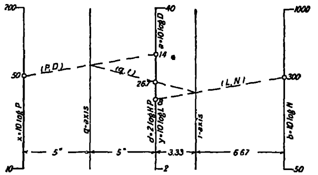


FIG. 31*a*.

(3) $H.P. = 0.000001983\, qt$ can be written $\log q + \log t = (\log H.P. - \log 0.000001983)$, which has the form (I). Our scales are

$$z = m_3 \log q = 5 \log q, \qquad c = m_6 \log t = 3.33 \log t, \qquad d = m_7 \log H.P.,$$

where $m_7 = m_3m_6/(m_3 + m_6) = 2$, and hence $d = 2 \log H.P.$ The z- and c-axes are 8.33 in. apart and the d-axis must divide this distance in the ratio $m_3 : m_6 = 5 : 3.33$; thus the d-axis must be at a distance of 5 in. from the z-axis and must coincide with the y- and a-axes. Thus, one side of this triple axis carries the scale for L and D, and the other side, the scale for $H.P.$ To get a starting point for the $H.P.$ scale we make a single computation: $P = 50, L = 14, D = 8, N = 300$ give $H.P. = 26.7$; hence the line joining $P = 50$ and $L = 14$ cuts the q-axis in a point, the line joining $D = 8$ and $N = 300$ cuts the t-axis in a point, and the line joining these two points cuts the d-axis in a point which must be marked $H.P. = 26.7$.

Fig. 31*b* gives the completed chart, and the index lines indicate that when $P = 50$ pounds per sq. in., $L = 14$ in., $D = 8$ in., and $N = 300$ revolutions per min., then $H.P. = 26.7$.

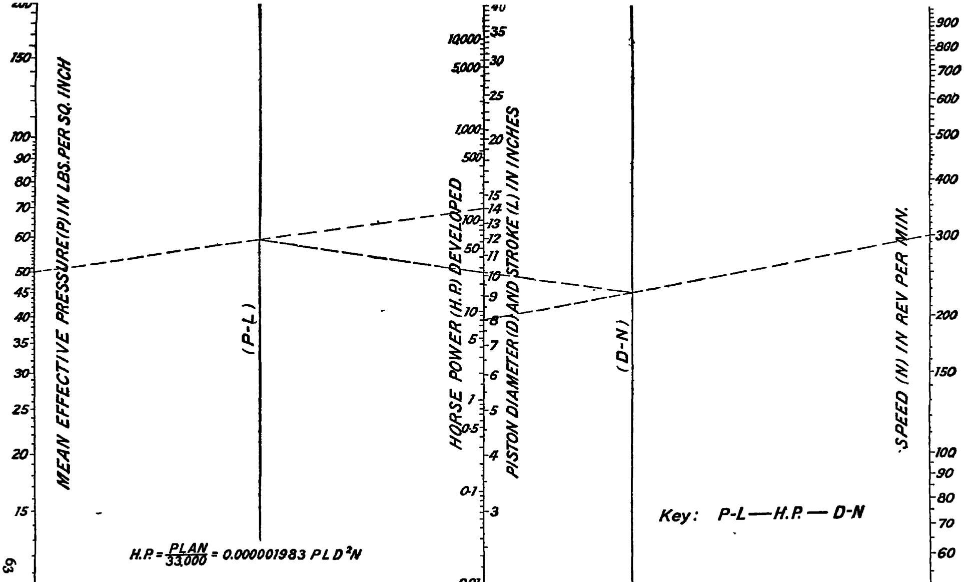

MEAN EFFECTIVE PRESSURE (P) IN LBS. PER SQ. INCH
(P-L)
HORSE POWER (H.P.) DEVELOPED
PISTON DIAMETER (D) AND STROKE (L) IN INCHES
(D-N)
SPEED (N) IN REV PER MIN.
Key: P-L—H.P.—D-N
H.P. = PLAN/33,000 = 0.000001983 P L D²N

FIG. 31b.

EXERCISES

Construct charts for the following formulas. The numbers in parenthesis suggest limiting values for the variables. These limits may be extended if necessary. Additional exercises will be found at the end of Chapter V.

1. $w = \frac{AP_1}{70} = \frac{\pi}{280} D^2P_1$. — Rankine's formula, for the weight, w, in pounds per sec. of steam flowing from a reservoir at pressure P_1 pounds (20 to 300) per sq. in. through an orifice of A sq in. or of diameter D in (0.1 to 2.0) to a pressure of P_2 pounds per sq. in., if $P_2 \leqq 0.6\ P_1$.

2. $Q = 3.33\, bH^{\frac{3}{2}}$. — Francis' formula for the discharge, Q, in cu. ft. per sec. over a rectangular weir b ft. (2 to 15) in width due to a head of H ft. (0.5 to 1.5) over the crest.

3. $L = 2 \ln \frac{d}{r} + 0.5$. — Self-inductance, L, in abhenries per cm. length of one of two parallel straight cylindrical wires each r cm. (0.1 to 0.25) in radius, their axes d cm. (2.5 to 144) apart, and conducting the same current in opposite directions [distance d small compared with length of wires].

4. $P = 50{,}210{,}000 \left(\frac{t}{D}\right)^3$. — Stewart's formula for the collapsing pressure, P, in pounds per sq. in. of Bessemer steel tubing t in. (0.02 to 0.13) in thickness and D in. (1 to 6) in external diameter.

5. $Q = \frac{2}{3}\sqrt{2g}\, cbH^{\frac{3}{2}}$ — Hamilton Smith formula for the discharge, Q, in cu. ft. per sec. over a contracted or suppressed weir b ft. (2 to 20) in width due to a head of H ft. (0.1 to 1.6) over the crest, if the coefficient of discharge is c (0 580 to 0 660). [$g = 32.2$.]

6. $P = 0.196 \frac{d^3}{r} f$. — Load, P, in pounds supported by a helical compression spring; d is the diameter of the wire in inches (0.102 to 0.460 or No 10 to No. 0000, B. S. gage), r is the mean radius of the coil in inches (0 5 to 2 0), f is the fiber stress in pounds (30,000 to 80,000).

7. $p = kgW\,(L + 10\,H)$. — Conveyor-belt calculations, p is the correct number of plies (1 to 15) W is the width of the belt in inches (10 to 60), g is the weight of material handled in pounds per cu. ft. (30 to 125), L is the length of the belt in ft. and H is the difference in elevation between the head and tail pulleys in ft. ($L + 10\,H$ 100 to 1500), k is a constant depending on the type of drive ($k = 1/250{,}000$ for a simple drive with bare pulley, $k = 1/300{,}000$ for a simple drive with rubber-lagged pulleys, $k = 1/375{,}000$ for a tandem drive with bare pulleys, $k = 1/455{,}000$ for a tandem drive with rubber-lagged pulleys) [Charted in Metallurgical and Chemical Engineering, Vol. XIV, Jan. 1, 1916]

8. $W = 15\,\pi d^2VD$. — Flow of steam through pipes; W is the weight of steam passing in pounds per min. (1 to 30,000), d is the inside diameter of the pipe in inches ($\frac{3}{4}$ to 36), V is the velocity of flow in ft. per sec. (15 to 250), D is the density of the steam at the mean pressure (use a steam table and plot D for values of the absolute pressure from 1 to 215 pounds per sq. in.) [Charted in Electrical World, Vol. 68, Dec. 9, 1916.]

9. $p = V^2DK$, where $K = \frac{0.0014}{d}\left(1 + \frac{3.6}{d}\right)$. — Flow of steam through pipes; p is the pressure drop between the ends of the pipe in pounds per sq. in per 100 ft. of pipe (0.01 to 20), and V, d, and D are defined in Ex. 8. [Charted in Electrical World, Vol. 68, Dec 9, 1916]

10. $p^2 - 14.7^2 = 0.0007 \frac{W^2}{D^4} H$. — Blast-pressure furnace; H is the height of the furnace in ft. (50 to 100), D is the bosh diameter in ft. (10 to 25), W is the number of cu. ft. of air at 70° F. per minute (5000 to 80,000), p is the blast-pressure in pounds gage (2 to 25) [Charted in Metallurgical and Chemical Engineering, Vol XIV, Mar. 15, 1916]

CHAPTER IV.

NOMOGRAPHIC OR ALIGNMENT CHARTS (*Continued*).

(III) EQUATION OF FORM $f_1(u) = f_2(v) \cdot f_3(w)$ or $f_1(u) = f_2(v)^{f_3(w)}$ — Z CHART.

32. **Chart for equation (III).** — [The second form of equation (III) can be brought immediately into the first form by taking logarithms of both members.] The first form of equation (III) is the same as the second form of equation (I), but in Art. 23 we used three parallel logarithmic scales, while here we shall use three natural scales, two parallel and a third oblique to them.

In Fig. 32*a*, let AX and BY be two parallel axes and AZ any axis oblique to these and cutting these in A and B respectively. Draw any

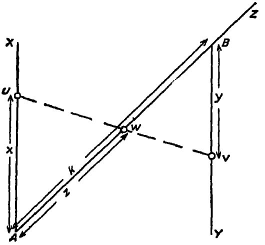


FIG. 32*a*.

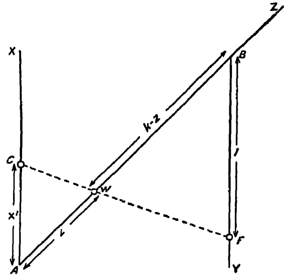


FIG. 32*b*.

index line cutting the axes in the points u, v, w so that $Au = x$, $Bv = y$, $Aw = z$; note that Au and Bv are oppositely directed. How are x, y, and z related?

Let $AB = k$. Then in the similar triangles Auw and Bvw,

$$Au : Bv = Aw : wB, \quad \text{or} \quad x : y = z : k - z, \quad \text{or} \quad x = \frac{z}{k - z}\, y.$$

Now if AX and BY carry the scales $x = m_1 f_1(u)$ and $y = m_2 f_2(v)$, the last equation becomes $f_1(u) = \dfrac{m_2 z}{m_1 (k - z)} f_2(v)$, and if AZ carries a scale for w such that

$$\frac{m_2 z}{m_1 (k - z)} = f_3(w) \quad \text{or} \quad z = k \frac{m_1 f_3(w)}{m_1 f_3(w) + m_2},$$

the equation becomes $f_1(u) = f_2(v) \cdot f_3(w)$, and any index line will cut the axes in three points corresponding to values u, v, w satisfying this equation.

Hence, *to chart equation* (III) $f_1(u) = f_3(w) \cdot f_2(v)$ *proceed as follows:* Draw three axes AX, BY, and AZ, where AX and BY are parallel and oppositely directed, and AB is any convenient length, k. With A and B as origins, construct on these axes the scales

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad z = k\frac{m_1 f_3(w)}{m_1 f_3(w) + m_2}.$$

Note that for the construction of the w-scale, it is necessary to compute the value of z for every value of w which is to appear on the chart. To avoid this computation, proceed as follows:

On BY, choose a fixed point F at any convenient distance, l, from B (Fig. 32*b*), and on AX construct the scale $AC = x' = l\frac{m_1}{m_2}f_3(w)$. From F as center, project the points C on the axis AZ. Let FC cut AZ in w, and let $Aw = z$. Then in the similar triangles ACw and BFw,

$$z : k - z = x' : l \quad \text{or} \quad z = \frac{kx'}{l + x'} = k\frac{m_1 f_3(w)}{m_1 f_3(w) + m_2}.$$

Hence to construct the scale $z = k\frac{m_1 f_3(w)}{m_1 f_3(w) + m_2}$, construct first the scale $x' = \frac{lm_1}{m_2} f_3(w)$ on AX, and then project this scale from the fixed point F on BY (where $BF = l$) to the axis AZ marking corresponding points with the same value of w.

This type of chart is illustrated in the following example.

33. Tension on bolts with U. S. standard threads, $D = 1.24\sqrt{\frac{L}{f_t}} + 0.088$. — Here D is the outside diameter of the bolt in inches, L is the load on the bolt in pounds, and f_t is the tension fiber stress in pounds per sq. in.

If we write the equation as $L = f_t\frac{(D - 0.088)^2}{(1.24)^2}$ we have an equation of the form (III). The scales are

$$x = m_1 L, \qquad y = m_2 f_t, \qquad x' = l\frac{m_1}{m_2}\frac{(D - 0.088)^2}{(1.24)^2}.$$

Let L vary up to 100,000 pounds; if we choose $m_1 = 0.0001$, the equation of the L-scale will be $x = 0.0001\,L$ and its length will be 10 in. Let f_t vary up to 100,000 pounds; if we choose $m_2 = 0.0001$, the equation of the f_t-scale will be $y = 0.0001\,f_t$ and its length will be 10 in. If we choose the fixed point or center of projection, F, on the y-axis so that $l = 8.3$ in., then the equations of our scales are

$$x = 0.0001\,L, \qquad y = 0.0001\,f_t, \qquad x' = 5.4\,(D - 0.088)^2.$$

If D varies from $\frac{1}{4}$ in. to 4 in., we compute the corresponding values of x' and lay off the scale on the x-axis. We then project this scale from the point F to the oblique axis, marking corresponding points with the same value of D (Fig. 33a).

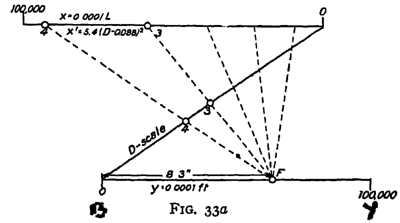


FIG. 33a

The final chart, showing neither the point F nor the projecting lines, is given in Fig. 33b. On one side of the oblique axis the threads per inch corresponding to the various diameters have been given. The index line indicates that when L = 20,000 pounds and f_t = 37,000 pounds per sq. in., then D = 1 in. and there are 8 threads to the inch.

Similar charts can be built up for various other threads.

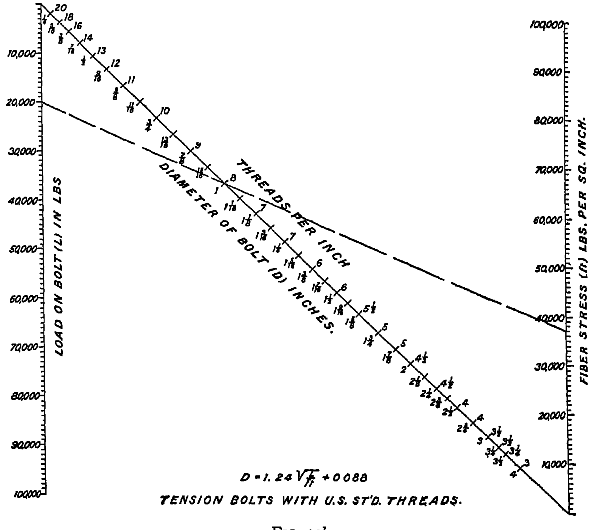


FIG. 33b.

(IV) EQUATION OF FORM $\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$ — TWO INTERSECTING INDEX LINES.

34. **Chart for equation (IV).** — A large number of equations involving four variables can be written in the form (IV) — such equations as $f_1(u) \cdot f_2(v) \cdot f_3(w) = f_4(t)$ or $f_1(u) \cdot f_2(v) = f_3(w) \cdot f_4(t)$, etc. Equation (IV) is included in the second form of equation (II), but in Art. 28 we used logarithmic scales whereas here we shall use natural scales.

Let AX, BY and AZ, BT be two pairs of parallel axes, where AZ may coincide with AX (Fig. 34*a*) or AZ may make any convenient angle with

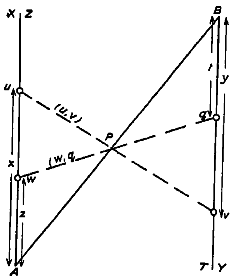


FIG. 34*a*.

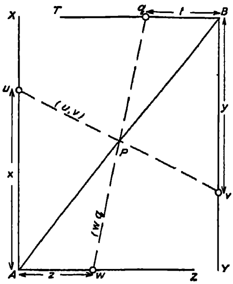


FIG. 34*b*.

AX (Fig. 34*b*), and where AB is a common transversal. Through any point P on AB draw two index lines cutting the axes in the points u, v, w, and q so that $Au = x$, $Bv = y$, $Aw = z$, and $Bq = t$. How are x, y, z, and t related?

From the similar triangles in these figures, we have

$$x : y = AP : PB \quad \text{and} \quad z : t = AP : PB, \qquad \therefore \quad x : y = z : t.$$

Now if AX, BY, AZ, BT carry the scales

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4 f_4(q),$$

where $m_1 : m_2 = m_3 : m_4$, the relation becomes $f_1(u) : f_2(v) = f_3(w) : f_4(q)$ and two index lines intersecting in a point on AB will cut out values of u, v, w, and q satisfying equation (IV).

Hence, to chart equation (IV) $f_1(u) \,.\, f_2(v) = f_3(w) : f_4(q)$ *proceed as follows*: Through the ends of a segment AB of any convenient length, draw the parallel axes AX and BY and the parallel axes AZ and BT, where AZ

may coincide or make any convenient angle with AX. On these axes construct the scales

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4 f_4(q)$$

where the moduli are arbitrary except for the relation $m_1 : m_2 = m_3 : m_4$. To read the chart, use two index lines, one joining u and v, and the other joining w and q, and intersecting in a point on AB.

The following examples illustrate this type of chart:

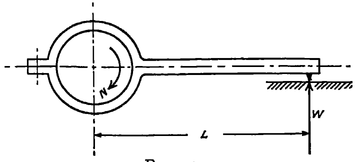


FIG 35*a*.

35. Prony brake or electric dynamometer formula, $H.P. = \dfrac{2\pi LNW}{33{,}000}$.—The sketch in Fig. 35*a* gives the method for measuring the power of a rotating shaft. Either the prony brake or the electric dynamometer may be used. With such an arrangement the power is given by the above formula, where L is the length of brake arm in feet, N is the speed of the shaft in revolutions per minute, and W is the load on scale in pounds.

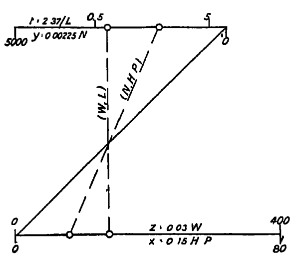


FIG. 35*b*.

If we write the equation as $\dfrac{H.P.}{N} = \dfrac{W}{5260/L}$, we have an equation of the form (IV), and our scales are

$$x = m_1 H.P., \qquad y = m_2 N,$$
$$z = m_3 W, \qquad t = m_4 \frac{5260}{L}.$$

The following table exhibits the limits of the variables, the choice of moduli, and the equations and approximate lengths of the scales.

Scale	*Limits*	*Modulus*	*Equation*	*Length*
$H.P.$	0 to 80	$m_1 = 0.15$	$x = 0.15\,H.P.$	12″
N	0 to 5000	$m_2 = 0.00225$	$y = 0.00225\,N$	$11\frac{1}{4}$″
W	0 to 400	$m_3 = 0.03$	$z = 0.03\,W$	12″
L	0 5 to 5	$m_4 = \dfrac{m_2 m_3}{m_1} = 0.00045$	$t = \dfrac{2.37}{L}$	4″

In Fig. 35*b*, the x- and z-axes coincide, so that the $H.P.$- and W-scales are laid off on opposite sides of the common axis, and starting from the

same origin; similarly for the N- and L-scales. The index lines, one joining L and W and the other joining N and $H.P.$ intersect on the transversal joining the zero points of the scales.

The completed chart is given in Fig. 35c, and the index lines show that when $L = 2$ ft., $W = 50$ pounds, and $N = 2000$ r.p.m., then $H.P. = 38$.

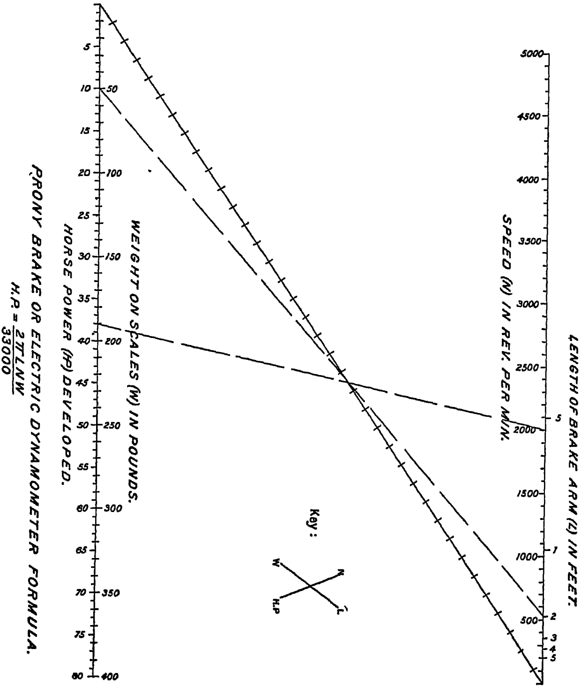


FIG. 35c.

36. Deflection of beam fixed at ends and loaded at center. $\Delta = \dfrac{WL^3}{192}\dfrac{1728}{EI}$.—Here, Δ is the deflection of beam in inches, W is the total load on beam in pounds, L is the length of beam in feet, E is the modulus

of elasticity of material in inch units, and I is the moment of inertia in inch units.

We shall take $E = 30{,}000{,}000$ for steel, so that the equation may be written as $\frac{\Delta}{L^3} = \frac{W}{3{,}333{,}000\, I}$, which has the form (IV), and gives the scales

$$x = m_1\Delta, \qquad y = m_2L^3, \qquad z = m_3W, \qquad t = m_4\,(3{,}333{,}000)\, I.$$

The following table exhibits the choice of moduli and the equations of the scales.

Scale	*Limits*	*Modulus*	*Equation*	*Length*
Δ	up to 1.5	$m_1 = 8$	$x = 8\,\Delta$	12″
L	10 to 35	$m_2 = 0.000{,}224$	$y = 0.000{,}224\, L^3$	10″
W	up to 300,000	$m_3 = 0.000{,}04$	$z = 0.000{,}04\, W$	12″
I	up to 3000	$m_4 = \frac{m_2m_3}{m_1} = 0.000{,}000{,}001{,}12$	$t = 0.003{,}735\, I$	11″

In Fig. 36a, the x- and z-axes are perpendicular and so are the y- and t-axes. The index lines, one joining W and I and the other joining Δ and L intersect on the common transversal joining the zero points of the scales.

The complete chart is given in Fig. 36b, and the index lines show that when $W = 130{,}000$ pounds, $I = 1000$ inch units, and $L = 25$ ft., then $\Delta = 0.61$ in.

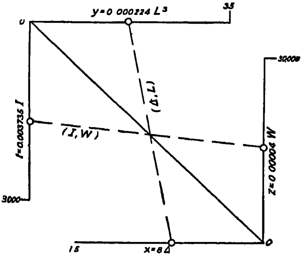
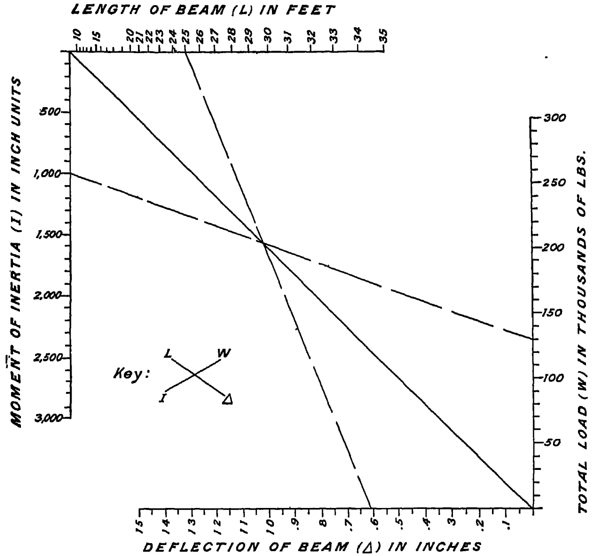


FIG. 36a.

37. Deflection of beams under various methods of loading and supporting. $\Delta = \frac{WL^3\,1728}{192\,aEI}$. — Here Δ is the deflection of the beam in inches, W is the total load on beam in pounds, L is the length of beam in feet, E is the modulus of elasticity of material in inch units, and I is the moment of inertia in inch units; a is a quantity whose value determines the method of loading and supporting, thus

(1) $a = 1$ — beam fixed at ends and loaded at center;
(2) $a = 2$ — " " " " " " uniformly;
(3) $a = \frac{1}{64}$ — " " " one end and loaded at other;
(4) $a = \frac{1}{24}$ — " " " " " " uniformly;
(5) $a = \frac{1}{4}$ — " supported at ends and loaded at center;
(6) $a = \frac{2}{5}$ — " " " " " " uniformly.

These six cases may be represented by six charts similar to those discussed in Arts. 35 and 36, for the equation can be written $\frac{\Delta}{L^3} = \frac{W}{3{,}333{,}000\,aI}$,



BEAM FIXED AT ENDS - LOADED AT CENTER.

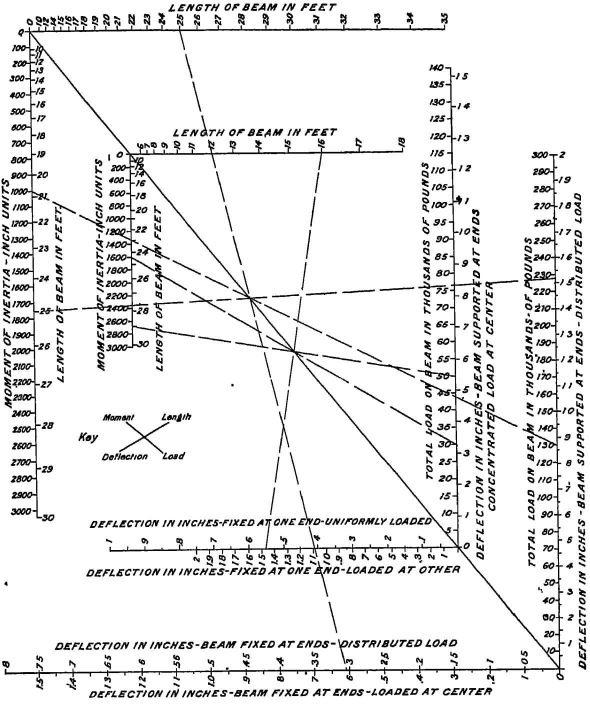
FIG. 36*b*.

which has the form (IV) when a value is assigned to a In all cases, the scales are

$$x = m_1\Delta, \qquad y = m_2L^3, \qquad z = m_3W, \qquad t = m_4\,(3{,}333{,}000)\,aI.$$

In cases (1) to (4), the x- and z-axes are perpendicular, and in cases (5) and (6), the x- and z-axes coincide. The scales are arranged so that there is only one common transversal joining the zeros of all the scales. In all cases the index line joining W and I and the index line joining Δ and L must intersect on the common transversal.

The completed chart, Fig. 37, clearly distinguishes the six cases so that there is no difficulty in reading it.



COMBINATION CHART-DEFLECTION OF BEAMS

Fig 37.

Another and more compact method of charting this composite equation will be given in Art. 43.

38. Specific speed of turbine and water wheel. $N_s = \dfrac{N\sqrt{H.P.}}{H^{\frac{5}{4}}}$. The formula gives the specific speed of a hydraulic reaction turbine

and also of a tangential water wheel Here, N_s is the specific speed, $H.P.$ is the horsepower, N is the number of revolutions per minute, and H is the head of water on turbine or wheel in ft. The formula is extensively used in Hydraulics and in water power engineering work; the reaction turbine is used when the head is low and the quantity of water available is relatively large, the value of N_s varying from 10 to 100, while the tangential water wheel is used when the head is great and, as is usual in such cases, the water limited, the value of N_s varying from 2 to 6. Because of this difference in the range of N_s for the two cases, it is best to construct separate charts.

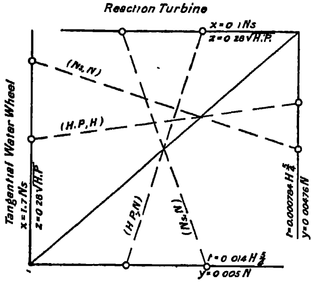


FIG. 38a.

If we write the equation as $\frac{N_s}{N} = \frac{\sqrt{H.P.}}{H^{\frac{5}{4}}}$, we have an equation of the form (IV), and our scales are

$$x = m_1 N_s, \qquad y = m_2 N, \qquad z = m_3\sqrt{H.P.}, \qquad t = m_4 H^{\frac{5}{4}}.$$

The following tables exhibit the choice of moduli and the equations of the scales.

Reaction Turbine

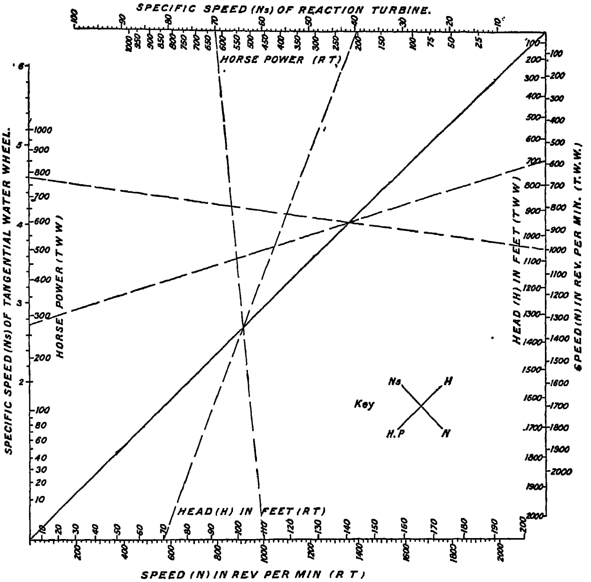
Scale	*Limits*	*Modulus*	*Equation*	*Length*
N_s	10 to 100	$m_1 = 0.1$	$x = 0.1\,N_s$	9″
N	up to 2000	$m_2 = 0.005$	$y = 0.005\,N$	10″
$H.P.$	" " 1000	$m_3 = 0.28$	$z = 0.28\sqrt{H.P.}$	9″
H	" " 200	$m_4 = \frac{m_2 m_3}{m_1} = 0.014$	$t = 0.014\,H^{\frac{5}{4}}$	10″

Tangential Water Wheel

Scale	*Limits*	*Modulus*	*Equation*	*Length*
N_s	2 to 6	$m_1 = 1.7$	$x = 1.7\,N_s$	7″
N	up to 2000	$m_2 = 0.00476$	$y = 0.00476\,N$	9.5″
$H.P.$	" " 1000	$m_3 = 0.28$	$z = 0.28\sqrt{H.P.}$	9″
H	" " 2000	$m_4 = \frac{m_2 m_3}{m_1} = 0.000784$	$t = 0.000784\,H^{\frac{5}{4}}$	10″

Fig. 38a shows the position of the scales. The x- and z-axes coincide so that the N_s- and $H.P.$-scales are constructed on opposite sides of the

common axis; similarly for the N- and H-scales. The charts for the reaction turbine and the tangential water wheel have been combined as shown in the diagram, *i.e*, the axes for the former have been placed perpendicular to the axes of the latter, and both charts use the same trans-



SPECIFIC SPEED OF TURBINE AND WATER WHEEL.

$$N_s = \frac{N\sqrt{HP}}{H^{5/4}}$$

FIG. 38*b*.

versal on which the index lines intersect, one index line joining N_s and N and the other joining $H\,P.$ and H.

Fig. 38*b* gives the completed chart; for the reaction turbine, the index lines show that when N = 1000 r.p.m., H = 70 ft., and $H.P.$ = 201, then N_s = 70; for the tangential water wheel, the index lines show that when N = 1000 r.p.m., H = 700, and $H.P.$ = 275, then N_s = 4.6.

(V). EQUATION OF FORM $f_1(u) = f_2(v) \cdot f_3(w) \cdot f_4(t) \ldots$ TWO OR MORE INTERSECTING INDEX LINES.

39. **Charts for equation (V).** — A large number of equations involving three or more variables can be written in the form (V), which is similar to the second form of equation (II), but in Art. 28 we used logarithmic scales, whereas here we shall use natural scales. Equation (IV) is a special case of equation (V) when there are four variables present. We shall here consider the cases where equation (V) contains three, five, or six variables. The method of charting to be employed is an amplification of the method described in Art. 34.

Case (1). *Three variables.* $f_1(u) = f_2(v) \cdot f_3(w)$. This equation can be written as $f_1(u) : f_2(v) = f_3(w) : 1$, which is of the form (IV); the scales are

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4,$$

where $m_1 : m_2 = m_3 : m_4$. Here the q-scale is replaced by a fixed point, P, on the y-axis and at a distance m_4 from B. The first index line joins u and v, the second index line joins w and the fixed point P; the two lines must intersect in a point on AB. (Figs. 34a, 34b.)

The fixed point, P, may be used as a center of projection from which the w-scale may be projected on the transversal AB. We shall then have two parallel scales and a third scale oblique to these, and a single index line will cut the scales in values of u, v, and w satisfying the equation. This method was employed in charting the formula for the tension on bolts in Art. 33.

An example illustrating case (1) is worked out in Art. 40.

Case (2). *Six variables.* $f_1(u) \cdot f_4(q) \cdot f_5(r) = f_2(v) \cdot f_3(w) \cdot f_6(s)$. This equation can be written as

$$f_1(u) : f_2(v) = f_3(w) : p \quad \text{and} \quad p : f_4(q) = f_5(r) : f_6(s).$$

Each of these equations has the form (IV) and can therefore be charted by the method described in Art. 34. In Fig. 39a, the p-, v-, and r-scales lie along a common axis, but the p-scale need not be graduated. To read the chart we need two pairs of index lines; the index lines (u, v) and (w, p) intersect in a point on AB, and the index lines (p, q) and (r, s) intersect in a point on BC.

An example illustrating case (2) is worked out in Art. 41.

Case (3). *Five variables.* $f_1(u) \cdot f_4(q) \cdot f_5(r) = f_2(v) \cdot f_3(w)$. This equation can be written as

$$f_1(u) : f_2(v) = f_3(w) : p \quad \text{and} \quad p : f_4(q) = f_5(r) : 1$$

and can be considered as a special form of case (2), where the s scale (Fig. 39a) is replaced by a fixed point through which the fourth index line must pass. An illustrative example is worked out in Art. 42.

We may also chart the equation $f_1(u) : f_2(v) = f_3(w) : p$ by the method described in Art. 34, and the equation $p = f_4(q) \cdot f_5(r)$ by the first method described in Art. 32. The arrangement of the scales is shown in Fig. 39*b*, and this arrangement is more compact than that of Fig. 39*a*, and employs only three index lines instead of four.

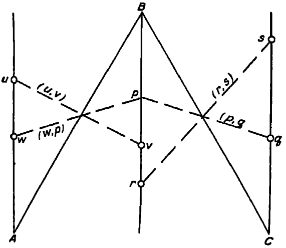


FIG. 39*a*.

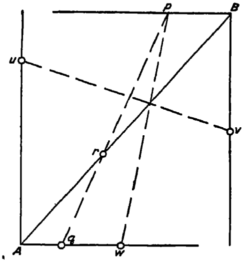


FIG. 39*b*.

In Fig. 39*b*, the *r*-scale lies along the transversal *AB* and the *q*- and *w*-scales are carried on the same axis; the index lines (u, v) and (w, p) intersect on the transversal *AB*, and the third index line aligns *p*, *r*, and *q*. An illustrative example will be found in Art. 43.

40. Twisting moment in a cylindrical shaft, $M = 0.196\, FD^3$. — Here *F* is the maximum fiber stress in pounds per sq. in., *D* is the diameter of the shaft in inches, and *M* is the twisting moment in inch pounds. If we write the equation as $M : D^3 = F : 5.1$ we have an equation of the form (V), case (1). Our scales are

$$x = m_1 M, \quad y = m_2 D^3,$$
$$z = m_3 F, \quad t = m_4\,(5.1).$$

The following table exhibits the choice of moduli and the equations of the scales:

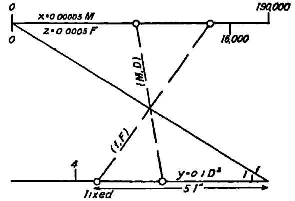
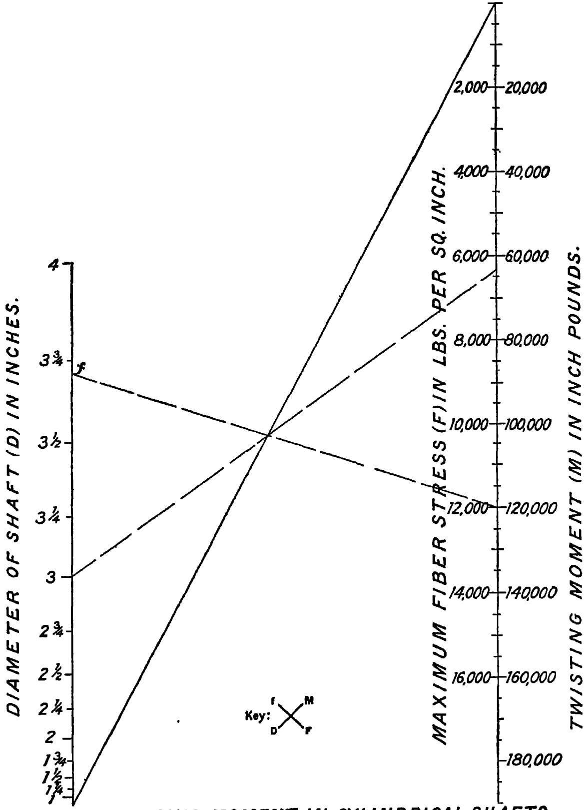


FIG. 40*a*.

Scale	*Limits*	*Modulus*	*Equation*	*Length*
M	up to 190,000	$m_1 = 0.00005$	$x = 0.00005\, M$	9.5″
D	1″ to 4″	$m_2 = 0.1$	$y = 0.1\, D^3$	6.4″
F	up to 16,000	$m_3 = 0.0005$	$z = 0.0005\, F$	8″



TWISTING MOMENT IN CYLINDRICAL SHAFTS

$M = 0.196\ F\ D^3$

FIG. 40*b*.

Now $m_4 = m_2 m_3 / m_1 = 1$, hence $t = 5.1$, and we have a fixed point on the y-axis at a distance 5.1 in. from the origin. We construct the M- and F-scales on the same axis from a common origin, and the D-scale and the fixed point on a parallel axis. (Fig. 40*a*.) The two index lines, one joining M and D and the other joining F and the fixed point, must intersect on the common transversal joining the zeros of the scales.

The completed chart is given in Fig. 40*b*, and the index lines show that when $F = 12{,}000$ pounds per sq. in. and $D = 3$ in., then $M = 63{,}500$ in. pounds.

41. D'Arcy's formula for the flow of steam in pipes, $P = \dfrac{B^2L}{c^2wd^5}$. — Here, P is the drop in pressure in pounds per sq. in., that is, the difference between the pressure, p_1, at the entrance to the pipe and the pressure, p_2, at the exit of the pipe; B is the weight of steam flowing in pounds per minute; L is the length of the pipe in feet; c is a quantity which varies with the nature of the inner surface of the pipe; w is the mean density of steam, *i.e.*, the average of the density at the entrance and the density at the exit of the pipe; d is the diameter of the pipe in inches. This formula is extensively used in engineering practice. We usually desire the pressure drop between two points. The chart to be constructed will however solve for any one of the six variables involved.

We have an equation involving six variables of the form (V), case (2), and as suggested in Art. 39, we shall separate it into two equations each involving four variables, and build up a Z chart for each of these. Taking the square root of both members of the equation, we write it

$$\sqrt{L}\,B = \sqrt{P}\,c\,\sqrt{w}\,\sqrt{d^5}, \quad \text{or} \quad \frac{\sqrt{L}\,B}{\sqrt{P}} = \frac{\sqrt{w}\,\sqrt{d^5}}{1/c}.$$

and equating both members to an auxiliary quantity, Q, we write

$$\frac{\sqrt{P}}{\sqrt{L}} = \frac{B}{Q}, \quad \text{and} \quad \frac{Q}{\sqrt{d^5}} = \frac{\sqrt{w}}{1/c}.$$

We now construct a Z chart for each of these equations, the two charts having the Q-axis in common.

For the first of these equations we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
P	0 to 25	$m_1 = 4$	$x = 4\sqrt{P}$	20″
L	0 to 1500	$m_2 = 0.4$	$y = 0.4\sqrt{L}$	16″
B	0 to 400	$m_3 = 0.02$	$z = 0.02\,B$	8″
Q		$m_4 = \dfrac{m_2 m_3}{m_1} = 0.002$		

The P- and B-scales (Fig. 41*a*) are placed on the same axis and starting from the same origin, and the L- and Q-scales on a parallel axis, but the Q-scale is not graduated.

For the second equation we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
Q		$m_4 = 0.002$		
d	0 to 10	$m_5 = 0.06$	$r = 0.06\sqrt{d^5}$	19″
w	0 to 10	$m_6 = 6$	$s = 6\sqrt{w}$	18″
c	30 to 70	$m_7 = \dfrac{m_5 m_6}{m_4} = 180$	$t = 180\left(\dfrac{1}{c}\right)$	6″

The w- and Q-scales are placed on the same axis; hence the w- and L-scales are on the common Q-axis. The d and c scales are placed on a parallel axis (Fig. 41*a*).

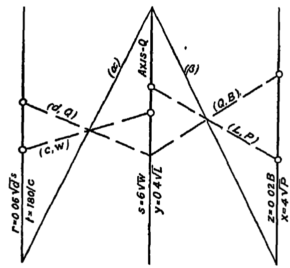


FIG. 41*a*.

We use four index lines. The (c, w) and (d, Q) lines must intersect on the common transversal of the corresponding scales, and the (Q, B) and (L, P) lines must intersect on the common transversal of the corresponding scales It is thus a simple matter to find the value of any one of the six variables when the other five are known. Thus, to find the value of P when c, w, d, B, and L are known, proceed as follows (Fig. 41*a*): join the point of intersection of (c, w) and the common transversal (α) with d, cutting the Q-axis in a point, Q; join the point of intersection of (Q, B) and the common transversal (β) with L, cutting out the required value of P.

Fig. 41*b* gives the completed chart, and the index lines show that when $c = 40$, $w = 2$, $d = 7$ in., $B = 300$ pounds per minute, and $L = 800$ feet, then $P = 1.34$ pounds per sq. in

42. Distributed load on a wooden beam. $F = \dfrac{9\,WL}{BH^2}$. — Here, F is the maximum fiber stress in pounds per sq. in.; L is the length of the beam in ft.; W is the total load on the beam in pounds; B is the width of the beam in inches; and H is the height of the beam in inches. In construction work, the total load on the beam (depending on the load which the floor must support), the allowable fiber stress (depending upon the kind and quality of the wood), and the length of the beam, are usually known; and the width and height of the beam are to be determined.

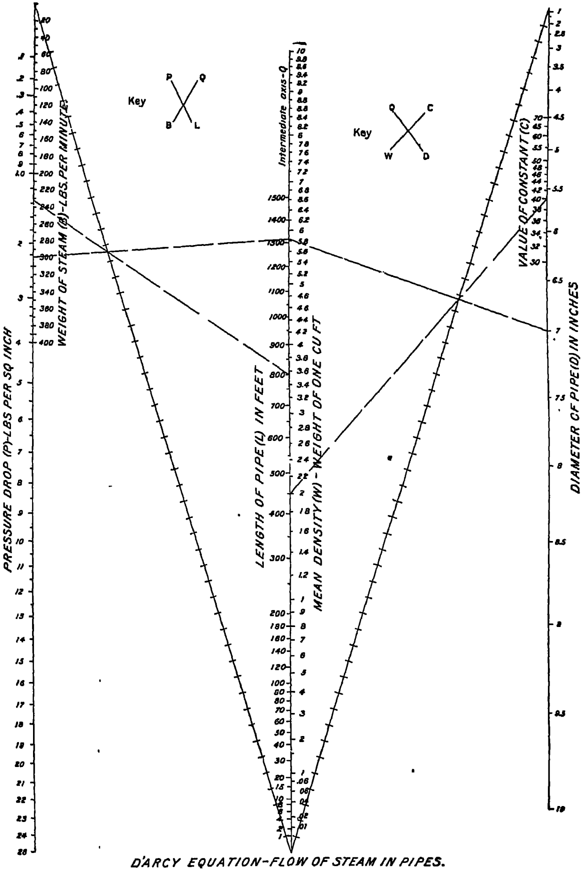

Key
P Q B L
Key
Q C W D
PRESSURE DROP (P)-LBS PER SQ INCH
WEIGHT OF STEAM (B)-LBS. PER MINUTE
Intermediate axis-Q
LENGTH OF PIPE (L) IN FEET
MEAN DENSITY (W) - WEIGHT OF ONE CU FT
VALUE OF CONSTANT (C)
DIAMETER OF PIPE (D) IN INCHES
D'ARCY EQUATION-FLOW OF STEAM IN PIPES.

FIG. 41*b*.

Since we have two unknown quantities we can of course get various combinations of these to satisfy the equation. By means of the chart to be constructed these combinations of width and height are readily seen, and any desired combinations may then be chosen.

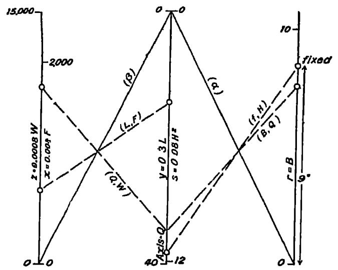


FIG. 42a.

We have an equation involving five variables of the form (V), case (3), and introducing an auxiliary quantity, Q, we shall separate it into two equations; thus,

$$\frac{F}{L} = \frac{W}{Q} \quad \text{and} \quad \frac{Q}{B} = \frac{H^2}{9}.$$

We now construct a Z chart for each of these equations, the two charts having the Q-axis in common.

For the first of these equations our scales are

$$x = m_1 F, \qquad y = m_2 (L), \qquad z = m_3 W, \qquad q = m_4 (Q),$$

and we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
F	up to 2000	$m_1 = 0.003$	$x = 0.003\,F$	6″
L	up to 40	$m_2 = 0.3$	$y = 0.3\,L$	12″
W	up to 15,000	$m_3 = 0.0008$	$z = 0.0008\,W$	12″
Q		$m_4 = \dfrac{m_2 m_3}{m_1} = 0.08$	$q = 0.08\,Q$	

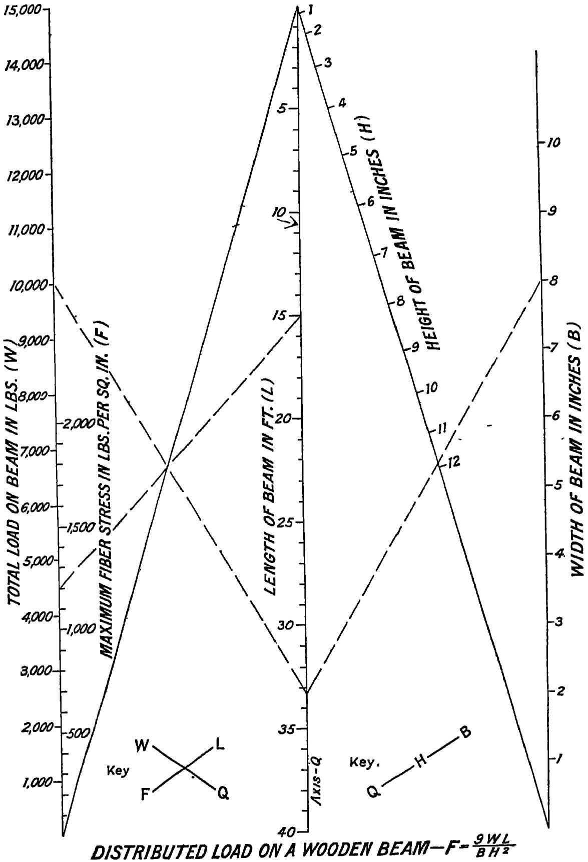
The F- and W-scales are placed on the same axis (Fig. 42a) and starting from the same origin, and the L- and Q-scales on a parallel axis, but the Q-scale is not graduated.

For the second equation our scales are

$$q = m_5 Q, \qquad r = m_6 B, \qquad s = m_7 H^2, \qquad t = m_8 (9),$$

and we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
Q		$m_5 = 0.08$	$q = 0.08\,Q$	
B	up to 10	$m_6 = 1$	$r = 1\,B$	10″
H	up to 12	$m_7 = 0.08$	$s = 0.08\,H^2$	11.5″
		$m_8 = \dfrac{m_6 m_7}{m_5} = 1$	$t = 9$	



DISTRIBUTED LOAD ON A WOODEN BEAM—$F = \frac{9WL}{BH^2}$

FIG. 42b.

The H- and Q-scales are placed on the same axis (Fig. 42a); hence the L- and H-scales are on the common Q-axis. The B-scale is placed on a parallel axis, on which there is also a fixed point, f, at a distance of 9.0 in. from the origin.

We use four index lines. The (L, F) and (Q, W) lines must intersect on the common transversal of the corresponding scales, and the (B, Q) and (f, H) lines must intersect on the common transversal of the corresponding scales. It is thus a simple matter to find the value of any one of the five variables when the other four are known. Thus to find the value of H when F, L, W, and B are known, proceed as follows: (Fig. 42a) join the point of intersection of (L, F) and the transversal (β) with W, cutting the Q-axis in a point, Q; join the point of intersection of (B, Q) and the transversal (α) with the fixed point, f, cutting out the required value of H.

If we wish we can project the H-scale on the transversal (α) using the fixed point, f, as a center of projection. We can then discard the fixed point, f, and the index line through it, for the index line (B, Q) will then cut the transversal (α) in the required value of H. Given then F, L, and W, we determine the point Q as above, and by rotating the index line through Q we can cut out any desired combination of B and H.

The completed chart is given in Fig. 42b, and the index lines show that when $W = 10{,}000$ pounds, $L = 15$ ft., $F = 1{,}200$ pounds per sq. in., and $B = 8$ in , then $H = 12$ in.

43. Combination chart for six beam deflection formulas. $\Delta = \dfrac{1728\, WL^3}{192\, EIP}$.—Here, W is the total load in pounds, L is the length of the beam in feet, I is the moment of inertia in inch units, Δ is the deflection in inches, E is the modulus of elasticity (30,000,000 for steel), and P is a factor which determines the method of loading and supporting. Thus when the beam is

(1) fixed at both ends and uniformly loaded,	$P = P_1 = 2$;
(2) fixed at both ends and loaded in center,	$P = P_2 = 1$;
(3) supported at both ends and uniformly loaded,	$P = P_3 = \frac{2}{5}$;
(4) supported at both ends and loaded in center,	$P = P_4 = \frac{1}{4}$;
(5) fixed at one end and uniformly loaded,	$P = P_5 = \frac{1}{24}$;
(6) fixed at one end and loaded at the other,	$P = P_6 = \frac{1}{64}$.

The equation thus involves five variables and is of the form (V), case (3). We introduce an auxiliary quantity, Q, and separate the equation into two equations; thus,

$$\frac{Q}{L^3} = \frac{W}{3{,}333{,}000\, I} \quad \text{and} \quad Q = \Delta P.$$

The first of these equations has already been charted in Art. 36, if we write Q for Δ; indeed Q is the deflection of a beam fixed at both ends and loaded in center, *i.e.*, for $P = 1$. We shall here use the same method of charting and the same scales employed in Art. 36. The scales are

$$x = m_1Q, \qquad y = m_2L^3, \qquad z = m_3W, \qquad t = m_4\,(3{,}333{,}000)\,I,$$

and the following table exhibits the choice of moduli:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
Q		$m_1=8$	$x=8\,Q$	
L	10 to 35	$m_2=0.000{,}224$	$y=0.000{,}224\,L^3$	10″
W	up to 200,000	$m_3=0\ 000{,}04$	$z=0.000{,}04\,W$	8″
I	up to 2000	$m_4=\dfrac{m_2m_3}{m_4}=0.000{,}000{,}00112$	$t=0.003{,}735\,I$	7.5″

The scales are arranged in the form of a rectangle (Fig. 43*a*); the L- and I-scales start from one vertex, B, and the W- and Q-scales start from the opposite vertex, A, but the Q-scale is not graduated. The two index lines, one joining W and I and the other joining L and Q must intersect on the transversal AB.

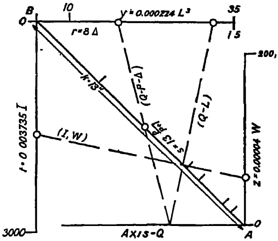


FIG. 43*a*.

We now chart the equation $Q = \Delta P$ by the method described in Art. 32 The scales are

$$x = m_1Q, \qquad r = m_2\Delta, \qquad s = k\,\frac{m_1P}{m_1P + m_2},$$

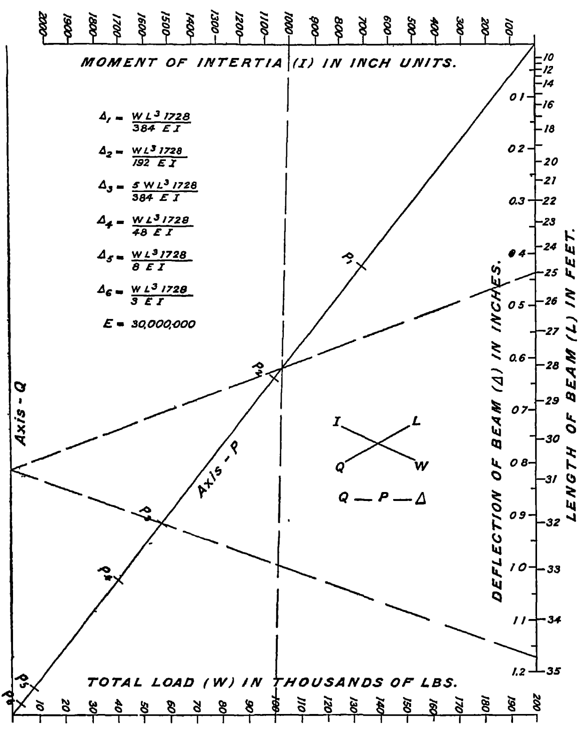
where the x- and r-axes must be parallel and extend in opposite directions, the s-axis is the transversal through the origins of these axes, and k is the distance between the origins. These conditions are met in Fig. 43*a* (where the x-axis is already constructed) if we make the r-axis coincide with the y-axis, and the s-axis with the transversal from A to B. We have drawn AB 13″ long, and we choose $m_2 = 8$, hence the equations of our scales are (Fig. 43*a*)

$$x = 8\,Q, \qquad r = 8\,\Delta, \qquad s = 13\,\frac{P}{P+1}.$$

The Δ- and L-scales are carried on opposite sides of their common axis. The six points $P_1, P_2, \ldots, P_6$ of the P scale are easily constructed by means of the table

P:	2	1	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{24}$	$\frac{1}{64}$
s:	8.67″	6.5″	3.7″	2 6″	0 52″	0.2″

To find the value of Δ when I, W, L, and P are known, proceed as follows: join the point of intersection of (I, W) and the transversal AB with



COMBINATION CHART – DEFLECTION OF BEAMS.

FIG. 43b.

L cutting the Q-axis in a point, Q; the line (Q, P) will cut out the required value of Δ.

Fig. 43b gives the completed chart, and the index lines show that

when $I = 1000$ inch units, $W = 100{,}000$ lbs., $L = 25$ ft., and $P = P_3$, *i.e.*, the beam is supported at both ends and uniformly loaded, then $\Delta = 1.17$ in.

44. General considerations. — All the equations charted thus far can be brought under the general forms

$$f_1(u) + f_2(v) + f_3(w) + \cdots = f_4(t) \quad \text{and} \quad f_1(u) \cdot f_2(v) \cdot f_3(w) \cdots = f_4(t).$$

Most of the formulas of engineering can be written in one of these forms. We have used various methods of charting these equations, employing logarithmic and natural scales. In the case of three variables, the underlying principle has been that one index line will cut the scales in three values satisfying the equation. In the case of four variables, the underlying principle has been that two index lines intersecting on an auxiliary axis will cut the scales in four values satisfying the equation; this method has been extended to equations involving more than four variables.

In the remainder of this chapter, we shall chart various forms of the above equations by methods requiring the use of parallel or perpendicular index lines. In Chapter V, we shall consider some equations which cannot be brought under either of the above forms, but which may be charted by methods requiring the use of parallel or perpendicular index lines or by methods involving the construction of curved axes. We shall end Chapter V with a brief discussion of various combined methods.

(VI) EQUATION OF FORM $f_1(u) : f_2(v) = f_3(w) : f_4(q)$. PARALLEL OR PERPENDICULAR INDEX LINES.

45. Chart of equation (VI). — Consider two pairs of intersecting axes AX, AY and BZ, BT so constructed that BZ is either parallel to or

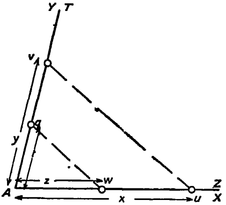


FIG. 45*a*.

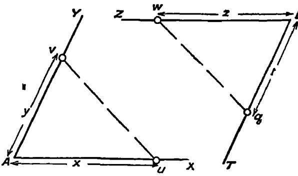


FIG. 45*b*.

coincides with AX and BT is either parallel to or coincides with AY, (Figs. 45*a*, *b*). Draw two parallel index lines, one meeting AX and AY, and the other meeting BZ and BT in u, v, w, and q respectively, so that

$Au = x$, $Av = y$, $Bw = z$, and $Bq = t$. Then, in the similar triangles uAv and wBt, we have $x : y = z : t$. Hence if AX, AY, BZ, BT carry the scales

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4 f_4(q),$$

respectively, where $m_1 : m_2 = m_3 : m_4$, then

$$x : y = z : t \quad \text{becomes} \quad f_1(u) : f_2(v) = f_3(w) : f_4(q),$$

which is equation (VI), and a pair of parallel index lines, (u, v) and (w, q) will cut out values of u, v, w, and q satisfying this equation. A pair of celluloid triangles will aid in reading the chart.

Consider again two pairs of intersecting axes AX, AY and BZ, BT so constructed that BZ is perpendicular to AX and BT is perpendicular to AY (Figs. 45c, d). Draw two perpendicular index lines, one meeting

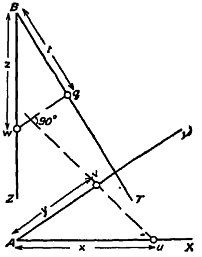


FIG. 45c.

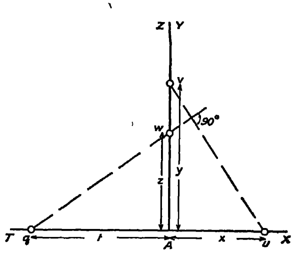


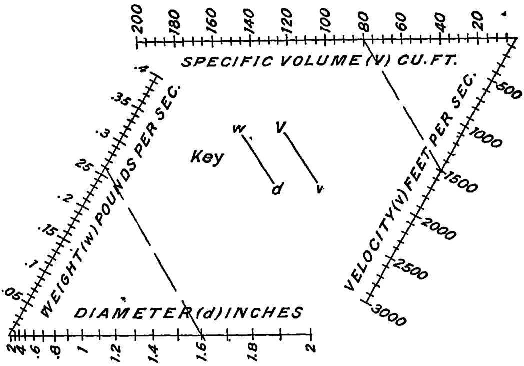
FIG. 45d.

AX and AY and the other meeting BZ and BT in u, v, w, and q respectively, so that $Au = x$, $Av = y$, $Bw = z$, and $Bq = t$. Then again $x : y = z : t$, and if our axes carry the scales described above, a pair of perpendicular index lines, (u, v) and (w, q), will cut out values of u, v, w, and q satisfying equation (VI). A sheet of celluloid with two perpendicular lines scratched on its under side will aid in reading the chart.

If the equation involves only three variables, *i.e.*, $f_1(u) = f_2(v) \cdot f_3(w)$, the equation can be written $f_1(u) : f_2(v) = f_3(w) : 1$; here the q-scale is replaced by a fixed point through which the second index line must always pass.

It is evident that there are other positions for the axes than those illustrated in Figs. 45a, b, c, d that will satisfy the conditions imposed by the problem.

46. Weight of gas flowing through an orifice. $w = \dfrac{\pi d^2 v}{576\,V}$. — Here, w is the weight of gas in pounds flowing per second, d is the diameter of the orifice in inches, v is the velocity of the gas in ft. per sec., and V is the specific volume in cu. ft. of the gas in the orifice.



DISCHARGE OF GAS THROUGH AN ORIFICE.

$$W = \frac{Av}{144\,V} = \frac{\pi d^2 v}{576\,V}.$$

FIG. 46.

If we write the equation $w : d^2 = v : 183.5\,V$, we have an equation of the form (VI). We shall build up a chart similar to that represented by Fig. 45*b*. The scales are

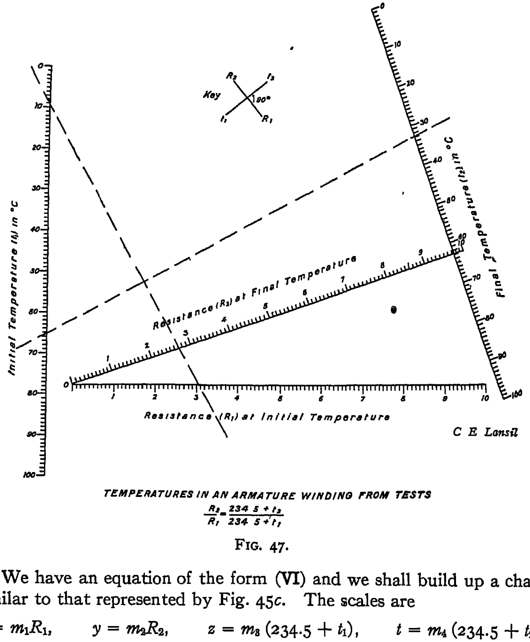
$$x = m_1 w, \qquad y = m_2 d^2, \qquad z = m_3 v, \qquad t = m_4\,(183.5\,V),$$

and the following table exhibits the choice of moduli:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
w	0 to 0.4	$m_1 = 10$	$x = 10\,w$	4″
d	0.2 to 2	$m_2 = 1$	$y = d^2$	4″
v	0 to 3000	$m_3 = 0.00135$	$z = 0.00135\,v$	4″
V	0 to 200	$m_4 = \dfrac{m_2 m_3}{m_1} = 0.000135$	$t = 0.02475\,V$	5″

The (w, d) and (v, V) index lines must be parallel. The chart is given in Fig. 46 and the index lines drawn show that when $v = 1500$ ft. per sec., $V = 80$ cu. ft., and $d = 1.6$ in., then $w = 0.26$ pounds per second.

47. Armature or field winding from tests. $\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}$.— Here R_1 and R_2 are resistances in ohms and t_1 and t_2 are the initial and final temperatures Centigrade in an armature or field winding.



TEMPERATURES IN AN ARMATURE WINDING FROM TESTS

$\frac{R_2}{R_1} = \frac{234.5 + t_2}{234.5 + t_1}$

FIG. 47.

We have an equation of the form (VI) and we shall build up a chart similar to that represented by Fig. 45*c*. The scales are

$$x = m_1 R_1, \qquad y = m_2 R_2, \qquad z = m_3 (234.5 + t_1), \qquad t = m_4 (234.5 + t_2),$$

and the following table exhibits the choice of moduli:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
R_1	0 to 10	$m_1 = 1$	$x = R_1$	10″
R_2	0 to 10	$m_2 = 1$	$y = R_2$	10″
t_1	0 to 100	$m_3 = 0.1$	$z = 23.45 + 0.1\, t_1$	10″
t_2	0 to 100	$m_4 = \frac{m_2 m_3}{m_1} = 0.1$	$t = 23.45 + 0.1\, t_2$	10″

We note that the points $t_1 = 0$ and $t_2 = 0$ are 23.45 in. from the point of intersection, B, of the z- and t-axes, which are respectively perpendicular to the x- and y-axes. But it is a simple matter to arrange the axes so that the t_1- and t_2-scales are within close range of the R_1- and R_2-scales. The (R_1, R_2) and (t_1, t_2) index lines must be perpendicular. The chart is given in Fig. 47, and the index lines drawn show that when $t_1 = 65°$, $t_2 = 33°$, and $R_1 = 3.04$ ohms, then $R_2 = 2.71$ ohms

48. Lamé formula for thick hollow cylinders subjected to internal pressure. $\dfrac{D^2}{d^2} = \dfrac{f+p}{f-p}$. — Here, D is the exterior diameter of the cylinder in inches, d is the interior diameter of the cylinder in inches, f is the fiber stress in pounds per sq. in., and p is the internal pressure in pounds per sq. in. The formula is extensively used in the design of thick pump and press cylinders. It is also used in ordnance work on big guns, to determine what is known as the elastic resistance curve of the steel at various sections of the gun from breech to muzzle.

We have an equation of the form (VI) and we shall build up a chart similar to that represented by Fig. 45*d*. The scales are

$$x = m_1 d^2, \qquad y = m_2 D^2, \qquad z = m_3 (f - p), \qquad t = m_4 (f + p),$$

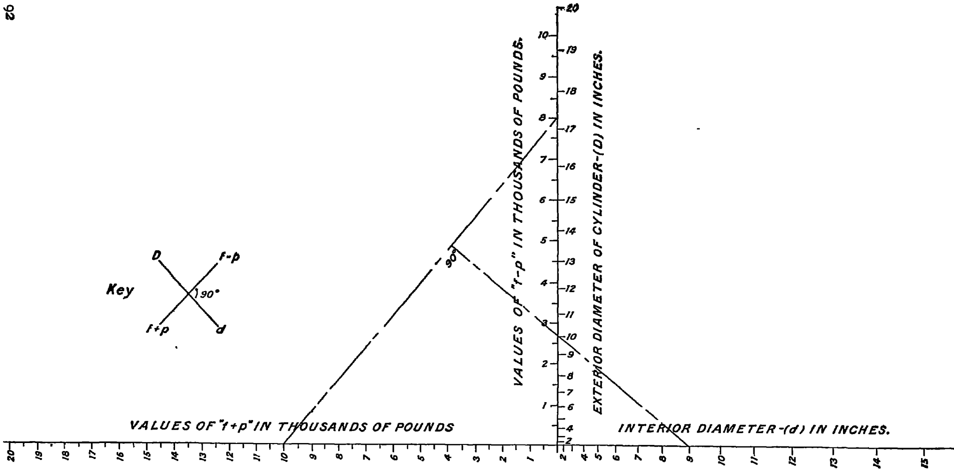
and the following table exhibits the choice of moduli:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
d	2 to 16	$m_1 = 0.03$	$x = 0.03\, d^2$	7.5″
D	2 to 20	$m_2 = 0.02$	$y = 0.02\, D^2$	8″
$f - p$	0 to 10,000	$m_3 = 0.00075$	$z = 0.00075\,(f - p)$	7.5″
$f + p$	0 to 20,000	$m_4 = \dfrac{m_2 m_3}{m_1} = 0.0005$	$t = 0.0005\,(f + p)$	10″

The (d, D) and $(f - p, f + p)$ index lines must be perpendicular. The chart is given in Fig. 48, and the index lines drawn show that when $f = 9000$ pounds per sq. in., $p = 1000$ pounds per sq. in., and $d = 9$ in., then $D = 10.1$ in.

(VII) EQUATION OF FORM $f_1(u) - f_2(v) = f_3(w) - f_4(q)$ OR $f_1(u) : f_2(v) = f_3(w) : f_4(q)$. PARALLEL OR PERPENDICULAR INDEX LINES

49. Chart for equation (VII). — The second form of equation (VII) can be immediately transformed into the first form by taking logarithms of both members of the equation. This second form of equation (VII) is the same as equation (VI), but we shall here use logarithmic scales in charting it.



LAMÉ FORMULA: THICK HOLLOW CYLINDERS SUBJECTED TO INTERNAL PRESSURE.

$$\frac{D^2}{d^2} = \frac{f+p}{f-p}$$

f = FIBER STRESS – LBS. PER SQ INCH.

p = INTERNAL PRESSURE – LBS. PER SQ. INCH.

FIG. 48.

Consider a pair of parallel axes AX and BY, k_1 inches apart, and another pair of parallel axes CZ and DT, k_2 inches apart, parallel to the first pair; AB and CD are also parallel. (Fig. 49*a*.) Draw two parallel index lines, one intersecting AX and BY and the other intersecting CZ

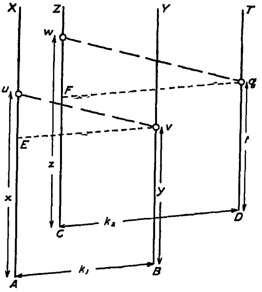


FIG. 49*a*.

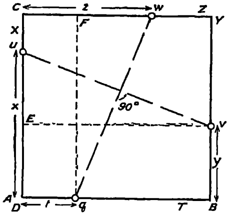


FIG. 49*b*.

and DT in u, v, w, and q respectively, so that $Au = x$, $Bv = y$, $Cw = z$, $Dq = t$. Draw vE and qF parallel to AB and CD respectively. Then in the similar triangles vEu and qFw, we have $x - y : k_1 = z - t : k_2$. Hence if AX, BY, CZ, DT carry the scales

$$x = m_1 f_1(u), \qquad y = m_1 f_2(v),$$
$$z = m_2 f_3(w), \qquad t = m_2 f_4(q),$$

where $m_1 : k_1 = m_2 : k_2$, then

$$x - y : k_1 = z - t : k_2$$

becomes

$$f_1(u) - f_2(v) = f_3(w) - f_4(q),$$

and a pair of parallel index lines, (u, v) and (w, q), will cut out values of u, v, w, and q satisfying this equation.

If CZ and DT are drawn perpendicular instead of parallel to Ax and By, and CD is perpendicular to AB (Fig. 49*b*), then a pair of perpendicular index lines, (u, v) and (w, q), will cut out values of u, v, w, and q satisfying the equation.

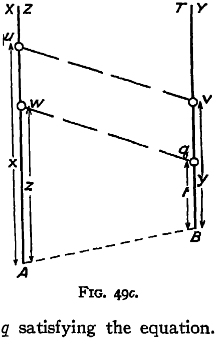


FIG. 49*c*.

To represent the equation $f_1(u) - f_2(v) = f_3(w) + f_4(q)$, the w- and q-scales must be laid off in opposite directions. If the axes are arranged in the form of a square, or if the second pair of axes coincide with the first pair (Fig. 49*c*) then $k_1 = k_2$; hence, $m_1 = m_2$ and all four scales have the

same modulus. Because of the restriction on the choice of moduli, this type of chart is not a very useful one. We shall only give a single illustration.

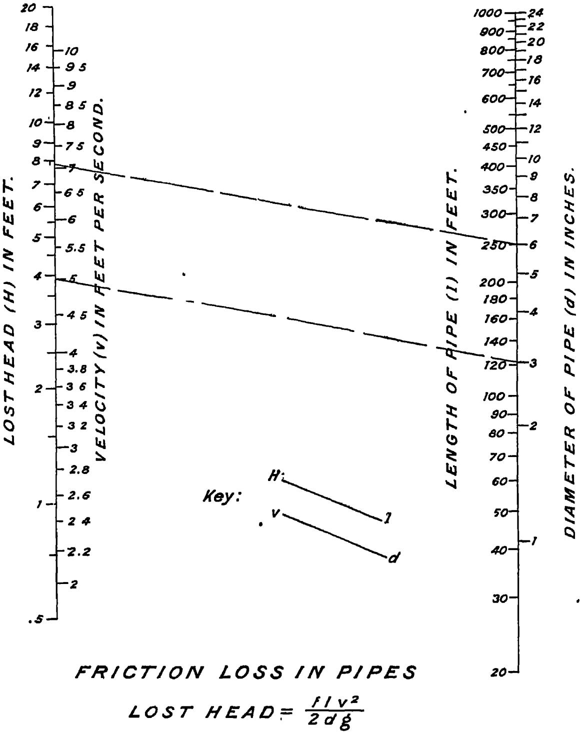


FIG. 50.

50. **Friction loss in flow of water.** $H = \dfrac{flv^2}{2\,dg}$. — Here l is the length of pipe in ft., v is the velocity in ft. per sec., d is the internal diameter of pipe in ft., H is the lost head in ft. due to friction, f is the friction factor, and $g = 32.2$.

If we replace g by 32.2, f by 0.02 (for clean cast-iron pipes) and express d in inches, our formula becomes

$$H = \frac{0.02\,lv^2\,(12)}{2\,(32.2)\,d}, \quad \text{or} \quad \frac{268.33\,H}{l} = \frac{v^2}{d},$$

or

$$(\log H + \log 268.33) - \log l = 2 \log v - \log d$$

an equation of the form (VII). We shall arrange the axes as in Fig. 49*c*. The scales are

$$x = m_1 (\log H + \log 268.33), \quad y = m_1 \log l, \quad z = m_1 (2 \log v), \quad t = m_1 \log d.$$

The following table exhibits the limits of the variables and the equations of the scales:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
H	0.5 to 20	$m_1 = 5$	$x = 5 \log H$	8″
l	20 to 1000	$m_1 = 5$	$y = 5 \log l$	8″
v	2 to 10	$m_1 = 5$	$z = 10 \log v$	7″
d	1 to 24	$m_1 = 5$	$t = 5 \log d$	7″

We lay off the l- and d-scales on opposite sides of a common axis and the v-scale on a parallel axis; these scales may start anywhere along these axes. We disregard the expression $m_1 \log 268.33$ in laying off the H-scale, and determine a starting point for this scale by making a single computation; thus, when $d = 3$, $l = 250$, and $v = 5$, then $H = 7.8$, and the index line through $l = 250$ drawn parallel to the index line joining $d = 3$ and $v = 5$ will cut the axis in a point which must be marked with the value $H = 7.8$. Thus the (H, l) index line is always parallel to the (v, d) index line.

The chart is given in Fig. 50, and the index lines drawn show that when $d = 3$ in., $l = 250$ ft., and $v = 5$ ft. per sec., then $H = 7.8$ ft.

EXERCISES

Construct charts for the following formulas. The numbers in parenthesis suggest limiting values for the variables. These limits may be extended if necessary. Additional exercises will be found at the end of Chapter V.

1. $B.H.P. = \frac{d^2m}{2.5}$. — Brake horse-power of an engine with m cylinders (2 to 12) of diameter d in. ($1\frac{1}{4}$ to 5), according to the rating of the Association of Automobile Manufacturers

2. $r_s = \frac{\pi}{4} d^2 f_s$. — Shearing strength, r_s, in pounds of a rivet d inches in diameter ($\frac{1}{8}$ to $\frac{7}{8}$) with an allowable stress in shear of f_s pounds per sq in (up to 15,000).

3. $M = 0.098\,fD^3$. — Bending moment, M, in inch-pounds on pins D inches in diameter (1 to 8) with an extreme fiber stress of f pounds per sq in (10,000 to 30,000). [It is better to build two charts, one for D varying from 1 to 3 and another for D varying from 3 to 8.]

4. $t = \frac{pd}{2f}$. — Thickness, t, in inches of a pipe of d inches internal diameter (0 to 60) to withstand a pressure of p pounds per sq. in. (0 to 100) with a fiber stress of f pounds per sq. in. (0 to 15,000).

5. $p = \frac{Rh}{V}$. — Approximate formula for flange rivets in a plate girder; h is the effective depth of the girder (20 to 110), V is the vertical shear in pounds (50,000 to 275,000), R is the rivet value in pounds (1000 to 20,000), p is the pitch of the rivets in inches (1 to 9).

6. $f = \frac{6M}{bh^2}$. — Intensity of stress, f, in pounds per sq. in. (750 to 1300) in the outer fiber of a rectangular beam, h inches in depth (3 to 20) and b inches in breadth (2 to 16) due to a bending moment of M inch-pounds.

7. $H = \frac{2\pi Ir^2}{d^3}$. — Field intensity, H, in lines per sq. cm. at a point on a line through the center and normal to the plane of a circular turn of wire of negligible section conducting a current of I abamperes (0 to 1000), the radius of the circular turn being r cm (4 to 12) and the distance of the point from the wire being d cm. $(4\sqrt{2}$ to $12\sqrt{2})$.

8. $C = \frac{wv^2}{rg}$. — Centrifugal force, w is the weight in pounds (1 to 150), v is the velocity in ft. per sec (1 to 50), r is the radius of the path in ft. (0.1 to 10), $g = 32.2$, C is the centrifugal force in pounds.

9. $P = wh\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)^2$. — Resistance to earth compression; w is the weight of the earth in pounds per cu. ft. (0 to 130), h is the depth in ft. (0 to 15), ϕ is the angle of repose of the earth (15° to 60°), P is the ultimate load on the earth in pounds per sq. ft. (0 to 35,000).

10. Apply the methods of this chapter to charting some of the formulas of the combination chart, Art. 25.

11. Apply the methods of this chapter to charting the formulas in Exercises 7, 8, and 9, at the end of Chapter III.

CHAPTER V.

NOMOGRAPHIC OR ALIGNMENT CHARTS (*Continued*).

(VIII) EQUATION OF FORM $f_1(u) + f_2(v) = \dfrac{f_3(w)}{f_4(q)}$. PARALLEL OR PERPENDICULAR INDEX LINES.

51. **Chart for equation (VIII).** — Consider two parallel axes, AX and BY, drawn in opposite directions, and two intersecting axes, AZ and AT, where AZ coincides with AX and AT coincides with the transversal AB. (Fig. 51*a*.) Draw two parallel index lines, one intersecting AX and BY and the other intersecting AZ and AT in u, v, w, and q respectively, so

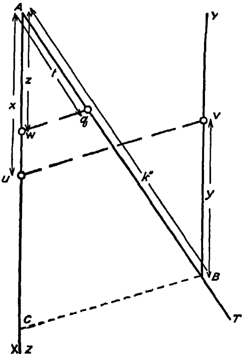


FIG. 51*a*.

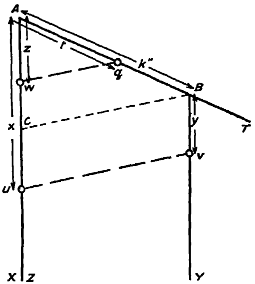


FIG. 51*b*.

that $Au = x$, $Bv = y$, $Aw = z$, $Aq = t$. Draw BC parallel to these index lines, and let $AB = k$ inches. Then in the similar triangles ACB and Awq, we have

$$AC : AB = Aw : Aq \quad \text{or} \quad x + y : k = z : t.$$

Now if AX, BY, AZ, and AT carry the scales

$$x = m_1 f_1(u), \qquad y = m_1 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4 f_4(q),$$

where $m_1 : k = m_3 : m_4$, then

$$x + y : k = z : t \quad \text{becomes} \quad f_1(u) + f_2(v) = f_3(w) : f_4(q),$$

and any pair of parallel index lines, (u, v) and (w, q), will cut the axes in values of u, v, w, q satisfying this equation. This type of chart is illustrated in Art. 52.

In Fig. 51*b*, AX and BY are drawn in the same direction, and hence $AC = x - y$, so that this arrangement serves to represent equation (VIII) when $f_1(u)$ and $f_2(v)$ are opposite in sign, or an equation of the form $f_1(u) - f_2(v) = f_3(w) : f_4(q)$.

In the construction of the chart for equation (VIII), we note the following: (1) The u-, w-, and q-scales are all laid off from the same origin, although we could have constructed AZ parallel to AX and AT parallel

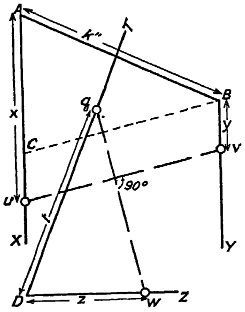


FIG 51*c*.

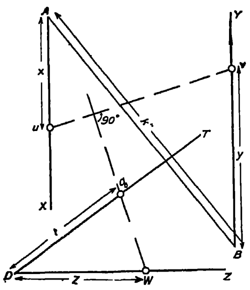


FIG 51*d*.

to AB without affecting the relations of the scales. (2) The u- and v-scales are constructed in opposite directions or in the same direction according as $f_1(u)$ and $f_2(v)$ have like or unlike signs. (3) The u- and v-scales have the same modulus, m_1, and the moduli and the length of the transversal, k, are connected by the relation $k = m_1m_4/m_3$. (4) The (u, v) and (w, q) index lines are always parallel.

If the equation (VIII) has the form $f_1(u) + f_2(v) = f_1(u) : f_4(q)$, containing only three variables, it can be charted in a similar manner. Here the w-scale coincides with the u-scale, so that the (w, q) index line coincides with the (u, v) index line; hence a single index line cuts the scales in values of u, v, and q satisfying the equation. This type is illustrated in Art. 53.

Consider again two parallel axes, AX and BY drawn in the same directions, and two intersecting axes DZ and DT, where DZ is perpendicular to AX and DT is perpendicular to the transversal AB (Fig. 51*c*). Draw two perpendicular index lines, one intersecting AX and BY and

the other intersecting DZ and DT in u, v, w, and q respectively, so that $Au = x, Bv = y, Dw = z, Dq = t$ Draw BC parallel to the first of these index lines. Then the triangles ACB and Dwq are similar (since their sides are mutually perpendicular). Hence

$$AC : AB = Dw : Dq, \quad \text{or} \quad x - y : k = z : t.$$

Now if AX, BY, DZ, and DT carry the scales

$$x = m_1 f_1(u), \qquad y = m_1 f_2(v), \qquad z = m_3 f_3(w), \qquad t = m_4 f_4(q),$$

where $m_1 : k = m_3 : m_4$, then

$$x - y : k = z : t \quad \text{becomes} \quad f_1(u) - f_2(v) = f_3(w) : f_4(q)$$

and any pair of perpendicular index lines, (u, v) and (w, q), will cut the axes in values of u, v, w, q satisfying this equation. This type is illustrated in Art. 54.

In Fig. 51*d*, AX and BY are drawn in opposite directions, and hence $AC = x + y$, so that this arrangement serves to represent equation (VIII) $f_1(u) + f_2(v) = f_3(w) : f_4(q)$.

52. Moment of inertia of cylinder. $I = \frac{W}{12}(3r^2 + h^2)$. — Here, W is the total weight of a right circular cylinder in pounds, r is the radius in inches, h is the height in inches, and I is the moment of inertia in inch units of the cylinder about an axis through its center of gravity and perpendicular to the axis of the cylinder.

Writing the equation as $3r^2 + h^2 = 12I : W$, we have an equation of form (VIII), and we shall follow Fig. 51*a* in the construction of the chart. Here

$$x = m_1(3r^2), \qquad y = m_1 h^2, \qquad z = m_3(12I), \qquad t = m_4 W.$$

Choosing $k = 15''$, we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
r	0 to 25	$m_1 = 0.008$	$x = 0.024\, r^2$	15″
h	0 to 40	$m_1 = 0.008$	$y = 0.008\, h^2$	13″
I	0 to 6,000,000	$m_3 = 0.000{,}000{,}2$	$z = 0.000{,}002{,}4\ I$	14″
W	0 to 25,000	$m_4 = \frac{m_3 k}{m_1} = 0.000{,}375$	$t = 0.000{,}375\ W$	9″

The (r, h) and (I, W) index lines must always be parallel. Fig. 52 gives the completed chart, and the index lines drawn show that when $r = 10$ in., $h = 30$ in., and $W = 20{,}000$ pounds, then $I = 20 \times 10^5$ inch units.

Choosing $k = 10$ in., we have the following table:

Scale	*Limits*	*Modulus*	*Equation*	*Length*
p	up to 6	$m_1 = 2$	$x = 2p$	12″
D	up to 2	$m_1 = 2$	$y = -2D$	4″
R	up to 200,000	$m_3 = 1.65$	$z = 0.00003R$	6″
t	up to 1	$m_4 = \dfrac{m_3 k}{m_1} = 8.25$	$t' = 8.25t$	8″

As in Fig. 51*c*, the p- and D-scales extend in the same direction, and the t' axis must be drawn perpendicular to the transversal joining the origins of the x- and y-axes. The (p, D) and (R, t) index lines are always perpendicular.

The complete chart is given in Fig. 54, and the index lines drawn show that when $p = 3$ in., $D = \frac{7}{8}$ in., and $t = \frac{1}{2}$ in., then $R = 58{,}500$ pounds.

(IX) EQUATION OF FORM $\dfrac{1}{f_1(u)} + \dfrac{1}{f_2(v)} = \dfrac{1}{f_3(w)}$ OR $\dfrac{1}{f_1(u)} + \dfrac{1}{f_2(v)} + \dfrac{1}{f_3(w)} + \cdots = \dfrac{1}{f_4(q)}$. THREE OR MORE CONCURRENT SCALES.

55. Chart for equation (IX). — Consider three concurrent axes AX, AY, and AZ (Fig 55*a*). Let any index line cut these axes in u, v, and w respectively, so that $Au = x$, $Av = y$, and $Aw = z$. Through w draw

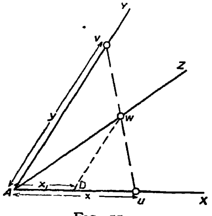


FIG 55*a*

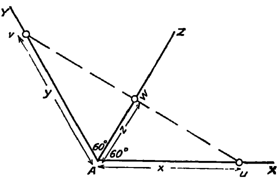


FIG. 55*b*.

wD parallel to AY and let $AD = x_1$. Let the position of AZ be determined by the ratio $AD : Dw = m_1 : m_2$. Then, in the similar triangles uDw and uAv, we have $Dw : Av = Du : Au$, or

$$\frac{m_2}{m_1} x_1 : y = x - x_1 : x, \quad \text{or} \quad \frac{m_1}{x} + \frac{m_2}{y} = \frac{m_1}{x_1}.$$

Now if AX carries the scales $x = m_1f_1(u)$, $x_1 = m_1f_3(w)$ and AY carries the scale $y = m_2f_2(v)$, then

$$\frac{m_1}{x} + \frac{m_2}{y} = \frac{m_1}{x_1} \quad \text{becomes} \quad \frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}$$

and any index line cuts out values of u, v, w satisfying this equation.

In the construction of the chart for equation (IX) we note the following: (1) The x- and y-axes may make any convenient angle with each other and they carry the scales $x = m_1f_1(u)$ and $y = m_2f_2(v)$. (2) The z-axis divides the angle between the x- and y-axes into two angles whose sines are in the ratio $m_1 : m_2$, *i.e.*, $AD : Dw = m_1 : m_2$. (3) The x-axis also carries the scale $x_1 = m_1f_3(w)$, and this scale is projected on the z-axis by lines parallel to the y-axis, the points and their projections being marked with the same value of w.

If $m_1 = m_2$, then AZ bisects the angle $XAY = \alpha$ (Fig. 55*b*). Then $Aw : AD = \sin(180° - \alpha) : \sin\frac{\alpha}{2}$, or

$$z = Aw = \frac{\sin\alpha}{\sin\frac{\alpha}{2}}x_1 = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}x_1 = m_1\left(2\cos\frac{\alpha}{2}\right)f_3(w).$$

In this case the w-scale may be constructed on AZ, and the scales are

$$x = m_1f_1(u), \qquad y = m_1f_2(v), \qquad z = m_1\left(2\cos\frac{\alpha}{2}\right)f_3(w).$$

Finally, if we take $\alpha = 120°$, our scales are simply

$$x = m_1f_1(u), \qquad y = m_1f_2(v), \qquad z = m_1f_3(w).$$

The method of charting the second form of equation (IX) is merely an extension of the method employed for charting the first form. Consider the case of four variables,

$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} + \frac{1}{f_3(w)} = \frac{1}{f_4(q)}.$$

By introducing an auxiliary variable, t, we can write

$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{t} \quad \text{and} \quad \frac{1}{t} + \frac{1}{f_3(w)} = \frac{1}{f_4(q)}.$$

We chart each of these equations by means of three concurrent scales with a common t-scale which need not be graduated (Fig. 55*c*.) Two index lines are necessary, one cutting the u- and v-scales and the other the w- and q-scales. The (u, v) and (w, q) index lines must intersect on the t-axis.

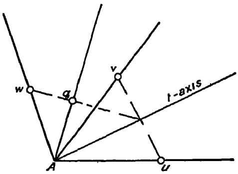


FIG. 55*c*.

Equations of the form (IX) are not very common in engineering practice. We shall only give one illustration.

56. Focal lengths of a lens. $\frac{1}{f}+\frac{1}{F}=\frac{1}{p}$. — Here, f is the focal distance of the object, F is the focal distance of the image, and p is the principal focal length of the lens.

We shall take our x- and y-axes at an angle of 120°, and the z-axis as the bisector of this angle. Let $m_1 = 0.5$, then the equations of our scales are

$$x = 0.5\,f, \qquad y = 0.5\,F, \qquad z = 0.5\,p.$$

The completed chart is given by Fig. 56. The index line drawn shows that when $f = 6$ and $F = 9$, then $p = 3.6$.

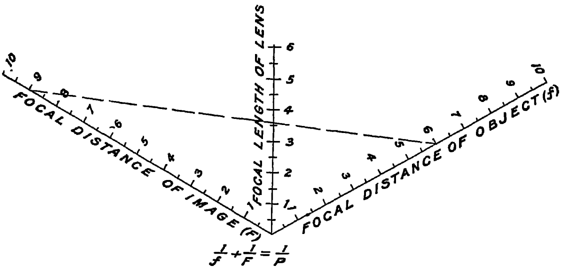


FIG. 56.

Another formula which may be charted in the same way is

$$\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots=\frac{1}{R}$$

where R is the circuit resistance of a circuit containing resistances R_1, R_2, R_3, . . . connected in parallel.

(X) EQUATION OF THE FORM $f_1(u)+f_2(v)\cdot f_3(w)=f_4(w)$. STRAIGHT AND CURVED SCALES

57. Chart for equation (X). — (We note that the variable w occurs in both members of the equation.) Consider two parallel axes AX and BY and a curved axis CZ (Fig. 57). Draw any index line cutting these axes in u, v, and w respectively. Draw wD parallel to AX, cutting AB in D, and draw wE and vF parallel to AB. The triangles uEw and wFv

are similar, hence $Eu : Fw = Ew : Fv = AD : DB$. Therefore, if $Au = x$, $Bv = y$, $AD = z_1$, $Dw = z$ and $AB = k$, we have

$$x - z : z - y = z_1 : k - z_1 \quad \text{or} \quad (k - z_1)\, x + z_1 y = kz,$$

or

$$x + \frac{z_1}{k - z_1}\, y = \frac{k}{k - z_1}\, z$$

and if

$$x = m_1 f_1(u), \qquad y = m_2 f_2(v), \qquad \frac{z_1}{k - z_1} = \frac{m_1}{m_2} f_3(w), \qquad \frac{kz}{k - z_1} = m_1 f_4(w),$$

this relation becomes $f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$. Solving for z_1 and z we get

$$z_1 = \frac{m_1 k}{m_1 f_3(w) + m_2} f_3(w),$$

$$z = \frac{m_1 m_2}{m_1 f_3(w) + m_2} f_4(w).$$

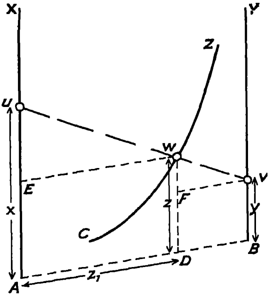


FIG. 57.

Hence to chart equation (**X**) proceed as follows: Construct the scales $x = m_1 f_1(u)$, $y = m_2 f_2(v)$ on two parallel axes AX and BY extending in the same direction. If $AB = k$ inches, construct the points of the curved scale CZ by assigning values to w, and laying off along AB, $z_1 = AD = \frac{m_1 k}{m_1 f_3(w) + m_2} f_3(w)$, and parallel to AX, $z = Dw = \frac{m_1 m_2}{m_1 f_3(w) + m_2} f_4(w)$, and marking the point thus found with the corresponding value of w. Then any index line will cut the three scales in values of u, v, and w satisfying equation (**X**).

To chart the equation $f_1(u) - f_2(v) \cdot f_3(w) = f_4(w)$, we construct the scales $x = m_1 f_1(u)$ and $y = -m_2 f_2(v)$ in opposite directions.

58. Storm water run-off formula. $q + Nq^{\frac{4}{5}} = P$. — This equation arises in the storm water run-off formula given by C. B. Buerger, in the Trans. Am. Soc. C. E., Vol. LXXVIII, p. 1139, where N and P are quantities which depend upon the sewer run, the area, and the rainfall, and q is the run-off in cu. ft. per sec. per acre.

If we write the equation $P - Nq^{\frac{4}{5}} = q$, we have an equation of the form (**X**), with the scales

$$x = m_1 P, \qquad y = -m_2 N, \qquad z_1 = \frac{m_1 k}{m_1 q^{\frac{4}{5}} + m_2} q^{\frac{4}{5}}, \qquad z = \frac{m_1 m_2}{m_1 q^{\frac{4}{5}} + m_2} q.$$

Let P, N, and q vary from 0 to 10, and take $m_1 = m_2 = 1$ and $k = 14''$.

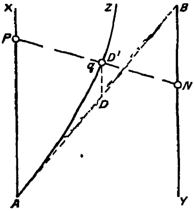


FIG. 58a.

Then our scales are

$$x = P, \quad y = -N, \quad z_1 = \frac{14\,q^{\frac{4}{5}}}{q^{\frac{4}{5}} + 1}, \quad z = \frac{q}{q^{\frac{4}{5}} + 1}.$$

The axes AX and BY are drawn in opposite directions, and the length of AB is 14 in. (Fig. 58*a*). We assign values to q, and on AB we lay off $AD = z_1$, and parallel to AX we lay off $DD' = z$ and mark the point D' with the value assigned to q. We join the points D' by a smooth curve, thus giving a curved scale for the variable q. Any index line will then cut out values of P, N, and q satisfying the equation.

The completed chart is given by Fig. 58*b*, and the index line drawn shows that when $P = 6$ and $N = 5$, then $q = 1$ cu. ft. per sec. per acre.

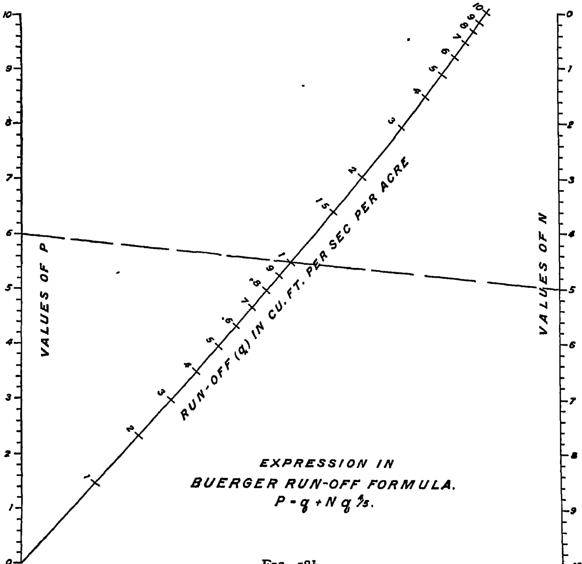


FIG. 58*b*.

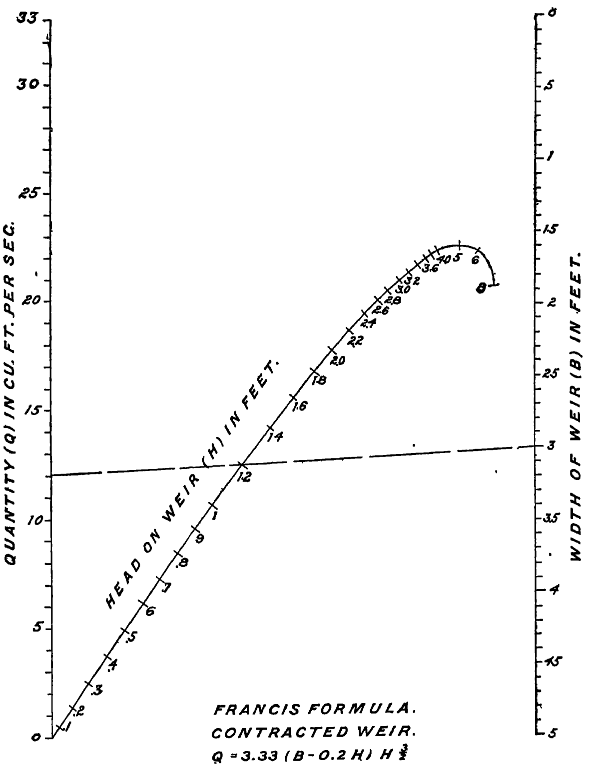

QUANTITY (Q) IN CU. FT. PER SEC.
HEAD ON WEIR (H) IN FEET.
WIDTH OF WEIR (B) IN FEET.
FRANCIS FORMULA.
CONTRACTED WEIR.
Q = 3.33 (B - 0.2 H) H 3/2

FIG. 59.

59. Francis formu a for a contracted weir. $Q = 3.33\,(B - 0.2\,H)\,H^{\frac{3}{2}}$. — Here, Q is the quantity of water flowing over weir in cu. ft. per sec., B is the width of the weir in ft., and H is the head over the crest of the weir in ft.

If we write the equation $Q - 3.33\,H^{\frac{3}{2}}B = -0.666\,H^{\frac{5}{2}}$ we have an equation of the form (**X**), with the scales $x = m_1Q$, $y = -m_2\,(3.33\,B)$, $z_1 = \dfrac{m_1kH^{\frac{3}{2}}}{m_1H^{\frac{3}{2}} + m_2}$, $z = -\dfrac{m_1m_2}{m_1H^{\frac{3}{2}} + m_2}\,(0.666\,H^{\frac{5}{2}})$. Let B vary from 0 to 5, H from 0 to 8, and Q from 0 to 33. If we choose $m_1 = 0.3$, $m_2 = 0.6$, and $k = 12$, our scales are $x = 0.3\,Q$, $y = -2\,B$, $z_1 = \dfrac{12\,H^{\frac{3}{2}}}{H^{\frac{3}{2}} + 2}$, $z = -\dfrac{0.4\,H^{\frac{5}{2}}}{H^{\frac{3}{2}} + 2}$.

The axes AX and BY are drawn in opposite directions, and the length of AB is 12 in. We assign values to H, and on AB we lay off $AD = z_1$, and parallel to BY we lay off $DD' = z$, and mark the point thus found with the corresponding value of H. We join the points by a smooth curve, thus giving a curved scale for the variable H. Any index line will then cut the scales in values of B, H, and Q satisfying the equation.

The completed chart is given in Fig 59, and the index line drawn shows that when $B = 3$ ft., and $H = 1.2$ ft., then $Q = 12.1$ cu. ft. per sec.

60. The solution of cubic and quadratic equations. —

$$w^3 + pw + q = 0, \qquad w^2 + pw + q = 0, \qquad w^3 + nw^2 + pw + q = 0.$$

Let us consider first the cubic equation $w^3 + pw + q = 0$. Writing the equation as $q + pw = -w^3$, we have an equation of the form (**X**). The scales are

$$x = m_1q, \qquad y = m_2p, \qquad z_1 = \frac{km_1}{m_1w + m_2}\,w, \qquad z = -\frac{m_1m_2}{m_1w + m_2}\,w^3.$$

If we allow p and q to vary from -10 to $+10$, and choose $m_1 = m_2 = 1$ and $k = 10''$, our scales are

$$x = q, \qquad y = p, \qquad z_1 = \frac{10\,w}{w + 1}, \qquad z = -\frac{w^3}{w + 1}.$$

In Fig. 60*a*, the p- and q-scales are constructed on XX' and YY' starting at A and B respectively. Assigning positive values to w, *viz.*, $w = 0, 0.1, 0.2, \ldots, 10$, we compute z_1 and z and lay off $AD = z_1$ and $DD' = z$, and mark the points, D', thus found with the corresponding values of w. We draw a smooth curve through these points, getting the curved axis AZ. Then any index line will cut the three scales in values of q, w, and p satisfying the equation, or an index line joining p and q will cut the curve in w, a root of the cubic equation.

We note that the line through A ($q = 0$) and D' ($w = w_0$) will cut YY' in E ($p = -w_0^2$) since these values of q, w, and p satisfy the equation $w^3 + pw + q = 0$. This observation allows us to construct the points of

ated. The position of the scales is illustrated in Fig. 63. The (u, q) and (v, w) index lines must intersect on the t-axis.

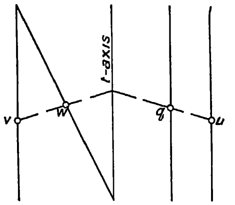


FIG. 62.

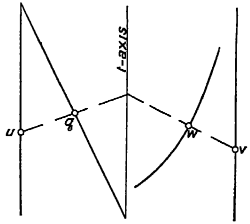


FIG 63.

64. Chart for equation of form $\frac{f_4(q)}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}$. — Introducing an auxiliary variable t, we write

$$(1)\ \frac{f_1(u)}{f_4(q)} = t \quad \text{and} \quad (2)\ \frac{1}{t} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}.$$

Equation (1) has the form (III) and may be plotted by the method of Art. 32. Equation (2) is of the form (IX) and may be plotted by the method of Art. 55. The t-scale is not graduated. Fig. 64 illustrates the position of the scales. The (u, q) and (v, w) index lines must intersect on the t-axis.

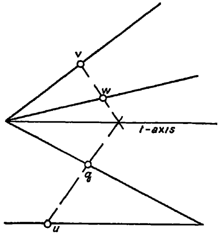


FIG. 64.

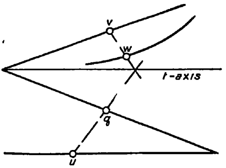


FIG. 65

65. Chart for equation of form $\frac{f_4(q)}{f_1(u)} + \frac{f_3(w)}{f_2(v)} = 1$. — Introducing an auxiliary variable t, we write

$$(1)\ \frac{f_1(u)}{f_4(q)} = t \quad \text{and} \quad (2)\ \frac{1}{t} + \frac{f_3(w)}{f_2(v)} = 1.$$

Equation (1) has the form (III) and can be plotted by the method of Art. 32. Equation (2) is a special case of the form charted in Art. 61. The common t-scale need not be graduated. The position of the scales is illustrated in Fig. 65. The (u, q) and (v, w) index lines must intersect on the t-axis.

66. Chart for equation of form $f_1(u) \cdot f_2(q) + f_3(v) \cdot f_4(w) = f_5(w)$. — We introduce a new variable t, and write

$$(1)\ f_1(u) f_2(q) = t \quad \text{and} \quad (2)\ t + f_3(v) f_4(w) = f_5(w).$$

Equation (1) has the form (III) and may be charted by the method of Art. 32. Equation (2) has the form (X) and may be charted by the method of Art. 57. The t-axis need not be graduated. The position of the scales is illustrated in Fig. 63. The (u, q) and (v, w) index lines must intersect on the t-axis.

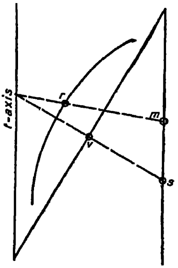


FIG. 66.

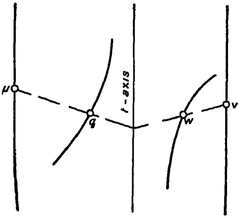


FIG. 67.

An interesting application of this type is given by D'Ocagne.* He considers Bazin's formula† for the velocity of flow of water in open channels,

$$v = c\sqrt{rs}, \quad \text{where} \quad c = \frac{87}{0.552 + \dfrac{m}{\sqrt{r}}}.$$

We can combine these equations into one equation,

$$v = \frac{87\sqrt{rs}}{0.552 + \dfrac{m}{\sqrt{r}}}, \quad \text{or} \quad \frac{87\sqrt{s}}{v} - \frac{m}{r} = \frac{0.552}{\sqrt{r}}.$$

Here $$\frac{87\sqrt{s}}{v} = t \quad \text{and} \quad t - \frac{m}{r} = \frac{0.552}{\sqrt{r}}.$$

* Traité de Nomographie, p. 233.

† We have charted this formula by means of two charts in Art. 51.

Fig. 66 illustrates the positions of the scales. By placing the m- and s-scales on the same axis, we get a very compact chart. The (v, s) and (m, r) index lines must intersect on the t-axis.

67. Chart for equation of form $f_1(u) \cdot f_2(q) + f_3(v) \cdot f_4(w) = f_5(q) + f_6(w)$. — We introduce an auxiliary variable t, and write

$$(1)\ f_1(u) \cdot f_2(q) - t = f_5(q) \quad \text{and} \quad (2)\ f_3(v)\ f_4(w) + t = f_6(w).$$

Both equations have the form (X) and can be plotted by the method of Art. 57 with a common t-axis, which need not be graduated. Fig. 67 illustrates the positions of the scales. The (u, q) and (v, w) index lines must intersect on the t-axis.

EXERCISES

Construct charts for the following formulas. The numbers in parenthesis suggest limiting values for the variables. These limits may be extended if necessary. Additional exercises will be found at the end of this chapter.

1. $V = \dfrac{\pi H}{9}\left(\frac{4}{5} D^2 + d^2\right)$. — Volume of a cask or buoy; d is the diameter of the base in ft. (0 to 10), D is the diameter of the middle section in ft. (0 to 10), H is the height in ft. (0 to 10), V is the volume in cu. ft. (0 to 800).

2. $Q = 3.33\, b\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right]$. — Francis' formula for the discharge, Q, in cu. ft. per sec. over a rectangular, suppressed weir b ft. in width (2 to 15) due to a head of H ft. over the crest (0.5 to 1.5), considering the velocity head h ft. (0 to 0.1) due to the velocity of approach.

3. $Sp.\ gr. = \dfrac{w}{w - w'}$. — Here, w is the weight in pounds of the solid in air (0 to 100), w' is the weight in pounds of the solid in water (0 to 95), $sp.\ gr.$ is the specific gravity (0 to 20).

4. $f = \dfrac{20{,}000}{1 + \dfrac{144\, L^2}{9000\, r^2}}$. — Gordon formula for columns with ends rounded and maximum allowable compression stress of 20,000 pounds per sq. in.; L is the length of the column in ft. (10 to 50), r is the radius of gyration in inches (1 to 12), f is the allowable stress in pounds per sq. in. (1000 to 20,000).

5. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$. — Equivalent resistance, R, of a parallel circuit the respective branches of which have resistances R_1, R_2, and R_3 ohms (1 to 10) and containing no e.m.f.

6. $s = v_0 t - \frac{1}{2} g t^2$. — Distance, s, in feet (−260 to +260) passed over by a body projected vertically upwards with an initial velocity of v_0 ft. per sec. (− 260 to + 260) in t seconds (0 to 17); $g = 32.2$.

7. $V = 0.6490\, \dfrac{T}{P} - \dfrac{22.58}{P^{\frac{1}{4}}}$. — Volume, V, in cu. ft. of one pound of superheated steam which has a pressure of P pounds per sq. in. (50 to 250) and a temperature of T degrees (280 to 650).

MISCELLANEOUS EXERCISES FOR CHAPTERS III, IV, V.

Construct charts for the following formulas. The numbers in parenthesis suggest limiting values for the variables. These limits may be extended if necessary.

1. $P' = \sqrt[3]{P_1^2P_2}$ and $P'' = \sqrt[3]{P_1P_2^2}$. — First intermediate pressure, P', in pounds per sq. in., and second intermediate pressure, P'', in pounds per sq in. of a three stage air compressor which compresses air from a pressure of P_1 pounds per sq. in. (14 4 to 15.2) to a pressure of P_2 pounds per sq. in. (500 to 3000).

2. $I = \frac{1}{2} Wr^2$. — Moment of inertia, I, in inch units of a right circular cylinder about its axis, W is the total weight in pounds (0 to 25,000) and r is the radius in inches (0 to 25).

3. $K = \sqrt{\dfrac{3r^2 + h^2}{12}}$. — Radius of gyration, K, of a right circular cylinder about an axis through its center of gravity and perpendicular to the axis of the cylinder; r is the radius in inches (0 to 25), h is the height in inches (0 to 40).

4. $C = \dfrac{k}{36 \ln \dfrac{d}{r} \times 10^5}$. — Capacitance, C, in microfarads of two parallel cylinders per cm. length; each cylinder r cm. in radius (0.1 to 0.25), their centers separated by a distance of d cm. (2.5 to 144), and immersed in a medium of dielectric constant k ($k = 1$ in practical cases).

5. $V = 0.596 \dfrac{T}{p} - 0.256$. — Volume, V, in cu. ft. of one pound of superheated steam which has a pressure of P pounds per sq. in. (50 to 250) and a temperature of T degrees (280 to 650).

6. $B.H.P. = 3.33 (A - 0.6 \sqrt{A})\sqrt{H}$. — Boiler horse-power, $B.H.P.$ (0 to 500) for chimney design for power houses; A is the internal area of chimney in sq. ft. (6 to 16), H is the height of the chimney in ft. (50 to 150).

7. $H.P. = \dfrac{nsd^3}{321{,}000}$. — Horse-power, $H.P$, transmitted by a solid circular shaft of diameter d in. (0.1 to 6) at n revolutions per min. (50 to 2500) with a fiber stress in shear of s pounds per sq. in. (0 to 50,000).

8. $K = \frac{1}{2} bc \sin A$. — Area of a triangle, K, with sides b (0 to 10) and c (1 to 10) and included angle A (0° to 90°).

9. $d = 0.013 \sqrt{Dlp}$. — Piston-rod diameter, d, in inches (1 to 6) of a steam engine; D is the piston diameter in inches (12 to 24), l is the length of the stroke in inches (12 to 60), p is the maximum steam pressure in pounds per sq. in. (80 to 150).

10. $A = 593 I \sqrt{\dfrac{ch}{c'p}}$. — Sectional area, A, in circular mils, of a copper wire for which the total annual cost of transmitting energy over a line conducting a constant current of I abamperes (0 to 100) will be a minimum; c is the cost of generated energy in dollars per kilowatt hour (0.005 to 0.02), c' is the cost of the bare copper wire in dollars per pound (0.15 to 0.35), h is the number of hours per year that the line is in use (4×300 to 24×300), p is the annual percentage rate of interest on the capital invested in copper (4 to 6).

11. $I = \dfrac{W}{12}(a^2 + b^2)$. — Moment of inertia, I, of a flat rectangular plate about an axis perpendicular to its plane and passing through the center, W is the total weight in pounds (0 to 30,000), a is the length in ft. (5 to 25), b is the breadth in ft. (2 to 10).

12. $T_1 = B \pm \sqrt{B^2 + T^2}$ or $T_1 - 2B = \dfrac{T^2}{T_1}$. — Bending moment, B, in a circular shaft for which the twisting moment is T, and T_1 is the twisting moment which would give the same effect as B and T acting together.

13. $p = \frac{st}{r+t}$. — Allowable internal pressure, p, in pounds per sq. in. on a hollow cylinder of inner radius r inches; t is the thickness of the cylinder in inches, s is the working strength of the material (20,000 pounds per sq. in. for steel).

14. $R_2 = R_1 [1 + \alpha (t_2 - t_1)]$. — Resistance, R_2, in ohms (0 to 5) of a conductor of t_2° C. which has a resistance of R_1 ohms (0 to 5) at t_1° C. (20 to 30) and is made of a material which has a resistance temperature coefficient of α at t_1° C. ($\alpha = 0.00393$ when $t_1 = 20°$ and may be taken as a constant for copper).

15 $Q = 3.31\, bH^{\frac{3}{2}} + 0.007\, b$. — Fteley and Stearns' formula for the discharge, Q, in cu ft per sec over a suppressed weir b feet in width (5 to 20) due to a head of H feet over the crest (0.1 to 1.6).

16. $D = H \tan A$. — Depth, D, in ft. (1 to 55,000) to a stratum, where A is the dip in degrees (1 to 89), and H is the horizontal distance in ft. (100 to 1,000).

17. $T = H \sin A$. — Thickness, T, in ft. (1 to 1,000), where A is the dip in degrees (1 to 90), and H is the horizontal distance in ft. (100 to 1,000)

18. $\tan C = \tan A \sin B$. — Projection of dips. C is the dip of the projected angle in degrees (0.1 to 89), A is the dip of the bed in degrees (1 to 89), B is the angle of projection in degrees (1 to 90).

19. $N = \frac{1}{4} R^3KC$. — Explosion formula. N is the number of half-pound blocks of T.N T. (20 to 10,000), R is the radius of rupture in ft. (0.5 to 15.0), K is a constant for the material (0.10 to 0 50), C is a constant for tamping (0.1 to 5.0)

20. $d^2 = 8\,rh - 4\,h^2$. — Diameter, d, of the base of a segment of a sphere of radius r and height of segment h.

21. $V = \pi rh^2 - \frac{\pi}{3} h^3$. — Volume, V, of a segment of a sphere of radius r and height of segment h.

22. $T = \frac{N}{\cos^3\alpha}$. — N is the number of teeth (1 to 100) in a spiral gear, α is the angle (0° to 80°) of teeth with axis, T is the number of teeth for which to select cutter (12–14, No 8; 14–17, No. 7, 17–21, No. 6; 21–26, No. 5; 26–35, No. 4; 35–55, No. 3; 55–135, No. 2; 135 up, No. 1)

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